Answers for Lesson 14-1 Exercises

1. $\cos\theta\cot\theta = \cos\theta\left(\frac{\cos\theta}{\sin\theta}\right) = \frac{1 - \sin^2\theta}{\sin\theta} = \frac{1}{\sin\theta} - \sin\theta$

2. $\sin\theta\cot\theta = \sin\theta\left(\frac{\cos\theta}{\sin\theta}\right) = \cos\theta$

3. $\cos\theta\tan\theta = \cos\theta\left(\frac{\sin\theta}{\cos\theta}\right) = \sin\theta$

4. $\sin\theta\sec\theta = \sin\theta\left(\frac{1}{\cos\theta}\right) = \frac{\sin\theta}{\cos\theta} = \tan\theta$

5. $\cos\theta\sec\theta = \cos\theta\left(\frac{1}{\cos\theta}\right) = 1$

6. $\tan\theta\cot\theta = \left(\frac{\sin\theta}{\cos\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right) = 1$

7. $\sin\theta\csc\theta = \sin\theta\left(\frac{1}{\sin\theta}\right) = \frac{\sin\theta}{\sin\theta} = 1$

8. $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{1}{\sin\theta}\cdot\frac{\cos\theta}{1} = \frac{1}{\sin\theta} \div \frac{1}{\cos\theta} = \frac{\csc\theta}{\sec\theta}$

9. 1 **10.** $\sin^2\theta$ **11.** $\tan^2\theta$

12. $-\cot^2\theta$ **13.** $\csc\theta$ **14.** $\sin\theta$

15. $\cos\theta$ **16.** 1 **17.** $\sin\theta$

18. 1 **19.** 1 **20.** 1

21. $\sec\theta$ **22.** 1 **23.** $\sec^2\theta$

24. $\sec^2\theta$ **25.** $\cot\theta$ **26.** $\csc\theta$

27. $-\tan^2\theta$ **28.** $\tan\theta$ **29.** $\sec\theta$

30. $\csc\theta$ **31.** $\sin^2\theta$ **32.** $\sin^2\theta$

33. $\sin\theta$ **34.** $\sec\theta$ **35.** $\sec\theta\csc^2\theta$

36. 1 **37.** 1 **38.** 1

39. $\pm\sqrt{1 - \cos^2\theta}$ **40.** $\frac{\pm\sqrt{1 - \cos^2\theta}}{\cos\theta}$

41. $\frac{\pm\sqrt{1 - \sin^2\theta}}{\sin\theta}$ **42.** $\pm\sqrt{1 + \cot^2\theta}$

43. $\pm\sqrt{\csc^2\theta - 1}$ **44.** $\pm\sqrt{1 + \tan^2\theta}$

45. $\sin^2\theta\tan^2\theta = \sin^2\theta\left(\frac{\sin^2\theta}{\cos^2\theta}\right) = (1 - \cos^2\theta)\left(\frac{\sin^2\theta}{\cos^2\theta}\right)$

$= \frac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta} = \frac{\sin^2\theta}{\cos^2\theta} - \frac{\sin^2\theta\cos^2\theta}{\cos^2\theta} = \tan^2\theta - \sin^2\theta$

46. $\sec\theta - \sin\theta\tan\theta = \frac{1}{\cos\theta} - \sin\theta\left(\frac{\sin\theta}{\cos\theta}\right)$

$= \frac{1}{\cos\theta} - \frac{\sin^2\theta}{\cos\theta} = \frac{1 - \sin^2\theta}{\cos\theta} = \frac{\cos^2\theta}{\cos\theta} = \cos\theta$

47. $\sin\theta\cos\theta(\tan\theta + \cot\theta) = \sin\theta\cos\theta\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$

$= \frac{\sin^2\theta\cos\theta}{\cos\theta} + \frac{\cos^2\theta\sin\theta}{\sin\theta} = \sin^2\theta + \cos^2\theta = 1$

48. $\frac{1 - \sin\theta}{\cos\theta} = \frac{1 - \sin\theta}{\cos\theta}\cdot\frac{\cos\theta}{\cos\theta} = \frac{(1 - \sin\theta)\cos\theta}{\cos^2\theta} =$

$\frac{(1 - \sin\theta)\cos\theta}{1 - \sin^2\theta} = \frac{(1 - \sin\theta)\cos\theta}{(1 - \sin\theta)(1 + \sin\theta)} = \frac{\cos\theta}{1 + \sin\theta}$

49. $\frac{\sec\theta}{\cot\theta + \tan\theta} = \frac{\frac{1}{\cos\theta}}{\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}}\cdot\frac{\sin\theta\cos\theta}{\sin\theta\cos\theta} =$

$\frac{\sin\theta}{\cos^2\theta + \sin^2\theta} = \frac{\sin\theta}{1} = \sin\theta$

50. $(\cot\theta + 1)^2 = \cot^2\theta + 2\cot\theta + 1 = \cot^2\theta + 1 + 2\cot\theta$
$= \csc^2\theta + 2\cot\theta$

51. $\frac{1 - \sin^2\theta}{\sin^2\theta}$

52. $1 - \sin\theta$

53. Check students' work.

54. When checking a root of an equation, you substitute the solution into the equation to see if it is true. When verifying an identity, you substitute equivalent expressions until both sides are the same.

55. $1 + \sec\theta = 1 + \frac{1}{\cos\theta} = \frac{1}{\cos\theta} + 1 = \frac{1 + \cos\theta}{\cos\theta}$

56. $\frac{1 + \tan\theta}{\tan\theta} = \frac{1}{\tan\theta} + \frac{\tan\theta}{\tan\theta} = \cot\theta + 1$

57. $\frac{\cot\theta\sin\theta}{\sec\theta} + \frac{\tan\theta\cos\theta}{\csc\theta} = \frac{\left(\frac{\cos\theta}{\sin\theta}\right)\sin\theta}{\frac{1}{\cos\theta}} + \frac{\left(\frac{\sin\theta}{\cos\theta}\right)\cos\theta}{\frac{1}{\sin\theta}} = \frac{\cos\theta}{\frac{1}{\cos\theta}}$
$+ \frac{\sin\theta}{\frac{1}{\sin\theta}} = \cos^2\theta + \sin^2\theta = 1$

58. $\sin^2\theta\tan^2\theta + \cos^2\theta\tan^2\theta = \tan^2\theta\,(\sin^2\theta + \cos^2\theta)$
$= \tan^2\theta\,(1) = \tan^2\theta = \sec^2\theta - 1$

59. 1

60. $\csc^2\theta$

61. If $n_2 > n_1$, then $\theta_1 > \theta_2$; if $n_2 < n_1$, then $\theta_1 < \theta_2$; if $n_2 = n_1$, then $\theta_1 = \theta_2$.

Answers for Lesson 14-2 Exercises

1. $-\frac{\pi}{2} + 2\pi n$ **2.** $0 + \pi n$ **3.** $\frac{\pi}{2} + 2\pi n$

4. $30° + n \cdot 360°$ and $150° + n \cdot 360°$

5. $30° + n \cdot 360°$ and $210° + n \cdot 360°$, or $30° + n \cdot 180°$

6. $210° + n \cdot 360°$ and $330° + n \cdot 360°$

7. $120° + n \cdot 360°$ and $300° + n \cdot 360°$, or $120° + n \cdot 180°$

8. $0.79 + \pi n$

9. $0.38 + 2\pi n$ and $2.76 + 2\pi n$

10. $-0.89 + 2\pi n$ and $4.04 + 2\pi n$

11. $1.89 + \pi n$

12. $2.67 + 2\pi n$ and $3.62 + 2\pi n$

13. no solution

14. $1.37 + \pi n$

15. $0.95 + 2\pi n$ and $5.33 + 2\pi n$

16. $\frac{\pi}{6}, \frac{5\pi}{6}$ **17.** $\frac{\pi}{6}, \frac{11\pi}{6}$ **18.** $\frac{\pi}{4}, \frac{5\pi}{4}$

19. $\frac{\pi}{4}, \frac{3\pi}{4}$ **20.** 0.84, 5.44 **21.** 0.46, 3.61

22. 0 **23.** 2.11, 5.25 **24.** no solution

25. $\frac{4\pi}{3}, \frac{5\pi}{3}$ **26.** $\frac{\pi}{2}, \pi, \frac{3\pi}{2}$ **27.** $\frac{\pi}{2}, \frac{3\pi}{2}$

28. $0, \pi, \frac{3\pi}{4}, \frac{7\pi}{4}$ **29.** $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ **30.** $0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}$

31. $0, \pi$ **32.** $0, \pi$ **33.** $\frac{7\pi}{6}, \frac{11\pi}{6}$

34. **a.** $t = \frac{2}{\pi} \cdot \cos^{-1}\left(\frac{h}{-4}\right)$
b. about 1.16 s, 1.33 s, 1.54 s
c. about 3.16 s, 3.33 s, 3.54 s

35. $30° + n \cdot 360°$ and $150° + n \cdot 360°$

36. $60° + n \cdot 360°$ and $300° + n \cdot 360°$

37. $210° + n \cdot 360°$ and $330° + n \cdot 360°$

38. $\frac{\pi}{3}, \frac{5\pi}{3}$ **39.** $\frac{3\pi}{2}$ **40.** 0.10, 3.24

41. 0.34, 2.80 **42.** 3.04, 6.18 **43.** 0

44. 0.0028 s; 0.019 s

45. **a.** $30° + n \cdot 360° \le x \le 150° + n \cdot 360°$

b. $150° + n \cdot 360° \le x \le 390° + n \cdot 360°$

c. Find the values of x where the graphs intersect, and then choose the appropriate interval.

46. $0 + 2\pi n, \frac{2}{3}\pi + 2\pi n, \frac{4}{3}\pi + 2\pi n$

47. $\frac{\pi}{2} + 2\pi n, \frac{3\pi}{2} + 2\pi n$

48. $\frac{\pi}{6} + 2\pi n, \frac{5\pi}{6} + 2\pi n, \frac{3\pi}{2} + 2\pi n$

49. $\frac{\pi}{4} + \frac{\pi}{2}n$

50. $\frac{\pi}{6} + 2\pi n, \frac{5\pi}{6} + 2\pi n, \frac{\pi}{2} + \pi n$

51. $0 + 2\pi n, \pi + 2\pi n, 1.25 + 2\pi n, 4.39 + 2\pi n$

52. $\frac{\pi}{2} + 2\pi n, \frac{7\pi}{6} + 2\pi n, \frac{11\pi}{6} + 2\pi n$

53. $\frac{\pi}{6} + \pi n, \frac{5\pi}{6} + \pi n$ **54.** $\frac{\pi}{6} + 2\pi n, \frac{5\pi}{6} + 2\pi n$

55. $1.107 + \pi n, \frac{3\pi}{4} + \pi n$

56. The student misinterpreted the meaning of $\cos^{-1} 0.5$ as being equal to $\frac{1}{\cos 0.5}$.

57. The student divided both sides of the equation by $\sin \theta$, which in the given interval can be equal to zero. Since division by zero is not possible, this is where the error was.

58. $2.09 + 2\pi n, 4.19 + 2\pi n$

59. $0.79 + \pi n, 2.36 + \pi n$

60. $0 + \pi n$

61. $0.79 + \pi n, 2.36 + \pi n$

62. $0 + \pi n, 0.79 + \pi n, 2.36 + \pi n$

63. $2.09 + 2\pi n, 4.19 + 2\pi n$

64. Answers may vary. Sample: In the first equation, you isolate the variable x to get the solution. In the trigonometric equation, you first isolate the trig. part, $\sin \theta$, but then you must continue to solve for θ.

65. $\frac{3\pi}{2} + 2\pi n$

66. **a.** Answers may vary. Sample: $\cos \theta = -1, 2 \cos \theta = -2, 3 \cos \theta = -3$

b. Start with $\cos \theta = -1$, and then multiply both sides of the equation by any nonzero number.

67. $\sin^{-1}\left(\frac{y}{2}\right)$

68. $\frac{1}{2} \cos^{-1}(y)$

69. $\sin^{-1}\left(\frac{y}{3}\right) - 2$

70. $\frac{1}{2\pi} \cos^{-1}\left(-\frac{y}{4}\right)$

71. $\cos^{-1}(y - 1)$

72. $\frac{1}{\pi} \cos^{-1}\left(\frac{y - 1}{2}\right)$

73. **a.** $K = 1250(\theta - \sin \theta)$ ft^2

b. ≈ 2.08

74. **a.** 1:55 A.M., 11:05 A.M., and 2:55 P.M.

b. 12:00 midnight to 1:55 A.M., 11:05 A.M. to 2:55 P.M.

Answers for Lesson 14-3 Exercises

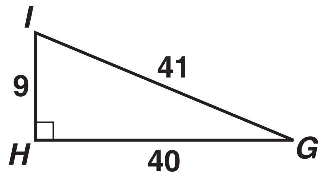
1. **a.** $\approx$8333 ft

b. $\approx$8824 ft

2. **a.** $\approx\frac{15}{17} \approx 0.88$ **b.** $\approx\frac{17}{8} \approx 2.13$ **c.** $\frac{8}{15} \approx 0.53$

d. $\frac{17}{8} \approx 2.13$ **e.** $\frac{17}{15} \approx 1.13$ **f.** $\frac{8}{15} \approx 0.53$

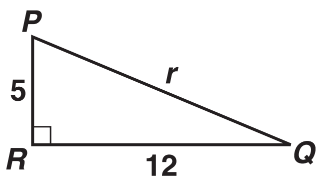
3.



a. $\frac{9}{41} \approx 0.22$ **b.** $\frac{40}{41} \approx 0.98$ **c.** $\frac{40}{9} \approx 4.44$

d. $\frac{41}{9} \approx 4.56$ **e.** $\frac{9}{41} \approx 0.22$ **f.** not defined

4.



$\sin P = \frac{12}{13} \approx 0.92,$

$\cos P = \frac{5}{13} \approx 0.38,$

$\tan P = \frac{12}{5} = 2.4,$

$\csc P = \frac{13}{12} \approx 1.08,$

$\sec P = \frac{13}{5} = 2.6$

5. 41.8 **6.** 25.2 **7.** 10.6

8. **a.** 300 ft

b. 445 ft

c. Answers may vary. Sample: The flagpole must be straight, the ground must be flat, and the flagpole and the ground must be perpendicular. You assume these things so that the flagpole and the ground form a right triangle. By having a right triangle you can use its properties to find the missing parts.

9. $45.0°$ **10.** $18.4°$ **11.** $48.6°$

12. $60.0°$ **13.** $19.6°$ **14.** $7.3°$

15. $74.3°$ **16.** $3.0°$ **17.** $68.0°$

18. $a \approx 8.7, m\angle A = 60.0°, m\angle B = 30.0°$

19. $c \approx 7.8, m\angle A \approx 39.8°, m\angle B \approx 50.2°$

20. $a = 9.0, m\angle A \approx 36.9°, m\angle B \approx 53.1°$

21. $c \approx 10.2, m\angle A \approx 52.6°, m\angle B \approx 37.4°$

22. $a \approx 8.0, m\angle A \approx 61.8°, m\angle B \approx 28.2°$

23. $b \approx 14.0, m\angle A \approx 50.6°, m\angle B \approx 39.4°$

24. **a.** $m\angle A = \cos^{-1}\left(\frac{1200}{d}\right)$
b. $37°$
c. $53°$

25. $\cos\theta = \frac{\sqrt{55}}{8}$,
$\tan\theta = \frac{3\sqrt{55}}{55}$,
$\csc\theta = \frac{8}{3}$,
$\sec\theta = \frac{8\sqrt{55}}{55}$,
$\cot\theta = \frac{\sqrt{55}}{3}$

26. $\sin\theta = \frac{3\sqrt{39}}{20}$,
$\tan\theta = \frac{3\sqrt{39}}{7}$,
$\csc\theta = \frac{20\sqrt{39}}{117}$,
$\sec\theta = \frac{20}{7}$,
$\cot\theta = \frac{7\sqrt{39}}{117}$

27. $\sin\theta = \frac{2\sqrt{6}}{5}$,
$\tan\theta = 2\sqrt{6}$,
$\csc\theta = \frac{5\sqrt{6}}{12}, \sec\theta = 5$,
$\cot\theta = \frac{\sqrt{6}}{12}$

28. $\sin\theta = \frac{24}{25}, \cos\theta = \frac{7}{25}$,
$\csc\theta = \frac{25}{24}, \sec\theta = \frac{25}{7}$,
$\cot\theta = \frac{7}{24}$

29. $\sin\theta = \frac{4}{7}, \cos\theta = \frac{\sqrt{33}}{7},$
$\tan\theta = \frac{4\sqrt{33}}{33},$
$\sec\theta = \frac{7\sqrt{33}}{33},$
$\cot\theta = \frac{\sqrt{33}}{4}$

30. $\sin\theta = \frac{5\sqrt{7}}{16}, \cos\theta = \frac{9}{16},$
$\tan\theta = \frac{5\sqrt{7}}{9},$
$\csc\theta = \frac{16\sqrt{7}}{35},$
$\cot\theta = \frac{9\sqrt{7}}{35}$

31. $\sin\theta = \frac{4\sqrt{41}}{41},$
$\cos\theta = \frac{5\sqrt{41}}{41}, \tan\theta = \frac{4}{5},$
$\csc\theta = \frac{\sqrt{41}}{4},$
$\sec\theta = \frac{\sqrt{41}}{5}$

32. $\cos\theta \approx 0.937, \tan\theta \approx 0.374, \csc\theta \approx 2.857, \sec\theta \approx 1.068,$
$\cot\theta \approx 2.676$

33. $\sin\theta \approx 0.192, \cos\theta \approx 0.981, \tan\theta \approx 0.196, \sec\theta \approx 1.019,$
$\cot\theta \approx 5.103$

34. **a.** $d = \frac{100}{\sin\theta}$
b. 115.5 ft, 130.5 ft

35. $a = 15, m\angle A \approx 61.9°, m\angle B \approx 28.1°$

36. $c \approx 12.2, m\angle A \approx 35.0°, m\angle B \approx 55.0°$

37. $a \approx 7.9, b \approx 6.2, m\angle B = 38°$

38. $a \approx 3.9, c \approx 6.9, m\angle B = 55.8°$

39. $a \approx 26.8, c \approx 28.1, m\angle A = 72.8°$

40. $a \approx 19.8, b \approx 2.9, m\angle A = 81.7°$

41. 35.5°

42. 33.4 ft

43. 20.3 m^2

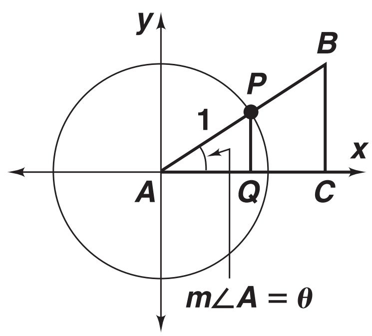
44. 136.2 ft; 46.6 ft

45. **a.** 12

b. 12

46. Using inverse sine, you can find that $\theta = 30°$. Since sine is positive in the first and second quadrants, another solution is 150°. All the solutions would be $30° + n \cdot 360°$ and $150° + n \cdot 360°$.

47.



Since ΔAPQ and ΔABC are similar triangles, $\frac{AQ}{AP} = \frac{AC}{AB}$.
So, $\cos\theta = AQ = \frac{AQ}{1} = \frac{AQ}{AP} = \frac{AC}{AB} = \cos A$.

48. **a.** ≈67 ft

b. ≈3.4°

49. $\sec A \stackrel{?}{=} \frac{1}{\cos A}$

$\frac{c}{b} \stackrel{?}{=} \frac{1}{\frac{b}{c}}$

$\frac{c}{b} = \frac{c}{b}$

50. $\tan A \stackrel{?}{=} \frac{\sin A}{\cos A}$

$\frac{a}{b} \stackrel{?}{=} \frac{\frac{a}{c}}{\frac{b}{c}}$

$\frac{a}{b} = \frac{a}{b}$

51. $\cos^2 A + \sin^2 A \stackrel{?}{=} 1$

$\left(\frac{b}{c}\right)^2 + \left(\frac{a}{c}\right)^2 \stackrel{?}{=} 1$

$b^2 + a^2 \stackrel{?}{=} c^2$

$c^2 = c^2$

52. **a.** 5.9 units

b. 31.8 units^2

c. 6.2 units^2

53. **a.** 72°

b. 19.0 cm

Answers for Lesson 14-4 Exercises

1. 18.7 cm^2	**2.** 9.1 $in.^2$	**3.** 81.9 m^2
4. 61.3 cm^2	**5.** 10.9	**6.** 9.2
7. 7.4	**8.** 12.2 in.	**9.** 10.7 m
10. 15.8 in.	**11.** 33.5°	**12.** 41.1°
13. 27.0°	**14.** 31.7°	**15.** 28.8°

16. **a.** $m\angle 1 = 4°, m\angle 2 = 58°$
b. 227 ft

17. $m\angle D = 100°, e = 22.3°, f = 34.2$

18. $m\angle A = 38.7°, m\angle B = 33.3°, b = 21.9$

19. $m\angle T = 29.3°, m\angle R = 26.7°, r = 8.3$

20. $m\angle B = 85°, b = 11.3, c = 8.7$

21. $m\angle C = 40°, b = 10, \mathrm{c} = 6.8$

22. $m\angle E = 62.8°, m\angle F = 82.2°, e = 17.1$ or $m\angle E \approx 47.2°$, $m\angle F \approx 97.8°, e \approx 14.1$

23. $m\angle F = 72°, e = 20$ in., $f = 23.5$ in.

24. $m\angle E = 40.3°, m\angle F = 85.7°, f = 12.3$ m

25. **a.** when $m\angle \mathrm{C} = 50°$
b. when $m\angle \mathrm{C} = 50°$
c. 90°; a right triangle has the greatest height.

26. Check students' work.

27. 7.5 mi, 7.9 mi	**28.** 32 cm	**29.** 66°
30. 85.0	**31.** 44.4	**32.** 29.7
33. 49.4	**34.** 267.3	**35.** 0.3
36. 8.5	**37.** 0.9	**38.** 8.2 m

39. 28.0 ft **40.** 43.2 yd **41.** 4.0 cm

42. 204 ft **43.** 2.4 miles

44. No; you need at least one side in order to set up a proportion you can solve.

45. **a.** 56.4°

b. 93.6°, 26.4°

c. No; ΔEFG could be congruent to ΔABC instead of ΔABD.

Answers for Lesson 14-5 Exercises

1. 37.1 **2.** 27.9 **3.** 13.7

4. 16.3 ft **5.** 10.0 cm **6.** 11.8 in.

7. 47.3° **8.** 27.0° **9.** 125.1°

10. 33.7° **11.** 83.3° **12.** 47.2°

13. 60.5° **14.** 50.8° **15.** 71.7°

16. 27.0° **17.** 35.5°

18. $b^2 = a^2 + c^2 - 2ac \cos B$

19. $c^2 = a^2 + b^2 - 2ab \cos C$

20. $\frac{\sin B}{b} = \frac{\sin C}{c}$

21. $\frac{\sin A}{a} = \frac{\sin B}{b}$

22. $\frac{\sin C}{c} = \frac{\sin A}{a}$

23. $a^2 = b^2 + c^2 - 2bc \cos A$

24. $b = 34.7, m\angle A = 26.7°, m\angle C = 33.3°$

25. $m\angle A = 50.1°, m\angle B = 56.3°, m\angle C = 73.6°$

26. $f = 6.2, m\angle D = 34.6°, m\angle E = 83.4°$

27. $m\angle A = 90°, m\angle B = 36.9°, m\angle C = 53.1°$

28. $g = 53.3, m\angle F = 31.9°, m\angle H = 38.1°$

29. $m\angle A = 56.1°, m\angle B = 70°, m\angle C = 53.9°$

30. **a–b.** Check students' work.

31. 46 ft

32. Assume one side is 1. Then use that side and its corresponding angle in the Law of Sines to find the ratio of the lengths of the other two sides.

33. 77.2°

34. **a.** 45.4 mi

b. 14.4° left; 4.4° west of north

35. 56.6° **36.** 11.0 cm **37.** 8.8 cm

38. 27.0° **39.** 117.3° **40.** 13.0 cm

41. 32.6° **42.** 21.5° **43.** 67.2°

44. 8.3 ft **45.** 64.0° **46.** 79.6°

47. 109 cm **48.** 18 cm

49. Yes; since $\cos 90° = 0$, $c^2 = a^2 + b^2 - 2ab \cos C$ reduces to $c^2 = a^2 + b^2$.

50. 4.8 in.

51. **a.** 2.1 m

b. 9.8 m^2

52. 3.0

Answers for Lesson 14-6 Exercises

1. $\csc\left(\theta - \frac{\pi}{2}\right) = \frac{1}{\sin\left(\theta - \frac{\pi}{2}\right)} = \frac{1}{\sin\left(-\left(\frac{\pi}{2} - \theta\right)\right)} = \frac{1}{-\sin\left(\frac{\pi}{2} - \theta\right)}$

$= \frac{1}{-\cos\theta} = -\sec\theta$

2. $\sec\left(\theta - \frac{\pi}{2}\right) = \frac{1}{\cos\left(\theta - \frac{\pi}{2}\right)} = \frac{1}{\cos\left(-\left(\frac{\pi}{2} - \theta\right)\right)} = \frac{1}{\cos\left(\frac{\pi}{2} - \theta\right)}$

$= \frac{1}{\sin\theta} = \csc\theta$

3. $\cot\left(\frac{\pi}{2} - \theta\right) = \frac{\cos\left(\frac{\pi}{2} - \theta\right)}{\sin\left(\frac{\pi}{2} - \theta\right)} = \frac{\sin\theta}{\cos\theta} = \tan\theta$

4. $\csc\left(\frac{\pi}{2} - \theta\right) = \frac{1}{\sin\left(\frac{\pi}{2} - \theta\right)} = \frac{1}{\cos\theta} = \sec\theta$

5. $\tan\left(\theta - \frac{\pi}{2}\right) = \tan\left(-\left(\frac{\pi}{2} - \theta\right)\right) = -\tan\left(\frac{\pi}{2} - \theta\right) = -\cot\theta$

6. $\sec\left(\frac{\pi}{2} - \theta\right) = \frac{1}{\cos\left(\frac{\pi}{2} - \theta\right)} = \frac{1}{\sin\theta} = \csc\theta$

7. $\frac{\pi}{2}, \frac{3\pi}{2}$

8. $\frac{\pi}{2}, \frac{3\pi}{2}$

9. $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

10. $\frac{\pi}{2}$

11. π

12. 2.034, 5.176

13. $\frac{\pi}{2}, \frac{3\pi}{2}$

14. $\frac{\pi}{2}$

15. $\sec A$

16. $\tan A$

17. 0

18. $\frac{\sqrt{2}}{2}$

19. -1

20. 0

21. $\frac{\sqrt{2} - \sqrt{6}}{4}$

22. $-\sqrt{3} - 2$

23. $2 - \sqrt{3}$

24. $\frac{\sqrt{2} + \sqrt{6}}{4}$

25. $\frac{\sqrt{6} - \sqrt{2}}{4}$

26. $2 + \sqrt{3}$

27. $-\frac{\sqrt{2}}{2}$

28. -1

29. $\frac{\sqrt{2} - \sqrt{6}}{4}$

30. $\frac{\sqrt{6} + \sqrt{2}}{4}$

31. $-2 + \sqrt{3}$

32. $-\frac{\sqrt{2}}{2}$

33. $-\frac{1}{2}$

34. $\frac{1}{2}$

35. $\frac{1}{2}$

36. $\frac{\sqrt{3}}{3}$

37. $\sin(A - B) = \sin(A + (-B))$

$= \sin A \cos(-B) + \cos A \sin(-B)$

$= \sin A \cos B - \cos A \sin B$

38. $\tan(A - B) = \tan(A + (-B))$

$= \dfrac{\tan A + \tan(-B)}{1 - \tan A \tan(-B)}$

$= \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

39. $\cos(A + B) = \cos(A - (-B))$

$= \cos A \cos(-B) + \sin \mathrm{A} \sin(-B)$

$= \cos A \cos \mathrm{B} - \sin \mathrm{A} \sin \mathrm{B}$

40. $\tan(A + B) = \tan(A - (-B))$

$= \dfrac{\tan A - \tan(-B)}{1 + \tan A \tan(-B)}$

$= \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

41. $\sin\left(x + \frac{\pi}{3}\right) + \sin\left(x - \frac{\pi}{3}\right)$

$= \sin x \cos\frac{\pi}{3} + \cos x \sin\frac{\pi}{3} + [\sin x \cos\frac{\pi}{3} - \cos x \sin\frac{\pi}{3}]$

$= \sin x \cos\frac{\pi}{3} + \cos x \sin\frac{\pi}{3} + \sin x \cos\frac{\pi}{3} - \cos x \sin\frac{\pi}{3}$

$= 2 \sin x \cos\frac{\pi}{3}$

$= 2 \sin x \cdot \frac{1}{2}$

$= \sin x$

42. $\sin\left(\frac{3\pi}{2} - x\right)$

$= \sin\frac{3\pi}{2} \cos x - \cos\frac{3\pi}{2} \sin \mathrm{x}$

$= -1 \cos x - 0 \cdot \sin \mathrm{x}$

$= -\cos x$

43. Answers may vary. Sample:

$\sin(30° + 60°) \stackrel{?}{=} \sin 30° + \sin 60°$

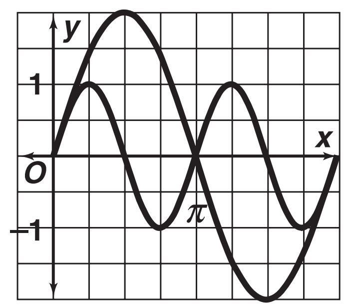
$\sin(90°) \stackrel{?}{=} \sin 30° + \sin 60°$

$1 \neq \frac{1}{2} + \frac{\sqrt{3}}{2}$

44. $\sin 3\theta$ **45.** $\sin 5\theta$ **46.** $\cos 7\theta$

47. $\cos 5\theta$ **48.** $\tan 11\theta$ **49.** $\tan 2\theta$

50. **a.**



b. No; it is not an identity since it is true for only a few specific x-values.

c. yes; $0 + 2\pi n$, $\pi + 2\pi n$, or $0 + \pi n$

d. Answers may vary. Sample: $\sin x = 2\sin(0.5x)$

51. $(5\cos\theta - 5\sqrt{3}\sin\theta, 5\sin\theta + 5\sqrt{3}\cos\theta)$

52. **a.** cosine, secant; sine, cosecant, tangent, and cotangent

b. No; answers may vary. Sample: $y = \sin(x + 0.5)$ is not odd because $y = \sin(-x + 0.5) = \sin(-(x - 0.5)) = -\sin(x - 0.5) \neq -\sin(x + 0.5)$.

53. $\cos(\pi - \theta) = \cos\pi \cdot \cos\theta + \sin\pi\sin\theta$
$= \cos\pi\cos\theta = -\cos\theta$

54. $\sin(\pi - \theta) = \sin\pi \cdot \cos\theta - \cos\pi\sin\theta$
$= -\cos\pi\sin\theta = \sin\theta$

55. $\sin(\pi + \theta) = \sin\pi \cdot \cos\theta + \cos\pi\sin\theta = -\sin\theta$

56. $\cos(\pi + \theta) = \cos\pi \cdot \cos\theta - \sin\pi\sin\theta = -\cos\theta$

57. Given a parallelogram with adjacent sides x_1 and x_2 and diagonals of d_1 and d_2, then by the Law of Cosines,

$d_1^2 + d_2^2 = x_1^2 + x_2^2 - 2x_1x_2 \cos \theta +$
$[x_1^2 + x_2^2 - 2x_1x_2 \cos(\pi - \theta)]$
$= 2x_1^2 + 2x_2^2 - 2x_1x_2 \cos \theta - 2x_1x_2$
$[\cos \pi \cos \theta + \sin \pi \sin \theta]$
$= 2x_1^2 + 2x_2^2 - 2x_1x_2 \cos \theta - 2x_1x_2 [-\cos \theta]$
$= 2x_1^2 + 2x_2^2 - 2x_1x_2 \cos \theta + 2x_1x_2 \cos \theta$
$= 2x_1^2 + 2x_2^2 = 2(x_1^2 + x_2^2)$

Answers for Lesson 14-7 Exercises

1. $-\frac{\sqrt{3}}{2}$

2. $-\frac{1}{2}$

3. $-\sqrt{3}$

4. 1

5. $-\frac{1}{2}$

6. $\sqrt{3}$

7. $-\frac{1}{2}$

8. $-\frac{\sqrt{3}}{2}$

9. $\sin 2\theta = \sin(\theta + \theta) = \sin\theta\cos\theta + \cos\theta \cdot \sin\theta$
$= 2\sin\theta\cos\theta$

10. $\tan 2\theta = \tan(\theta + \theta) = \frac{\tan\theta + \tan\theta}{1 - \tan\theta\tan\theta} = \frac{2\tan\theta}{1 - \tan^2\theta}$

11. $\frac{\sqrt{2 + \sqrt{3}}}{2}$

12. $\sqrt{7 - 4\sqrt{3}}$

13. $\frac{\sqrt{2 - \sqrt{3}}}{2}$

14. $\frac{\sqrt{2 - \sqrt{2}}}{2}$

15. $\frac{\sqrt{2 + \sqrt{2}}}{2}$

16. $\sqrt{3 - 2\sqrt{2}}$

17. 0

18. $\frac{\sqrt{2 - \sqrt{2 + \sqrt{3}}}}{2}$

19. $\frac{3\sqrt{10}}{10}$

20. $\frac{\sqrt{10}}{10}$

21. 3

22. $\frac{1}{3}$

23. $\frac{4\sqrt{17}}{17}$

24. $-\frac{\sqrt{17}}{17}$

25. -4

26. $-\sqrt{17}$

27. $\sin 2R = 2\sin R\cos R = 2\frac{r}{t} \cdot \frac{s}{t} = \frac{2rs}{t^2}$

28. $\cos 2R = \cos^2 R - \sin^2 R = \left(\frac{s}{t}\right)^2 - \left(\frac{r}{t}\right)^2 = \frac{s^2}{t^2} - \frac{r^2}{t^2} = \frac{s^2 - r^2}{t^2}$

29. $\sin 2S = 2\sin S\cos S = 2 \cdot \frac{s}{t} \cdot \frac{r}{t} = \frac{2sr}{t^2}$
$= 2\sin R\cos R = \sin 2R$

30. $\sin^2 \frac{S}{2} = \left(\sin\frac{S}{2}\right)^2 = \left(\pm\sqrt{\frac{1 - \cos S}{2}}\right)^2 = \frac{1 - \cos S}{2}$
$= \frac{1 - \frac{r}{t}}{2} = \frac{1}{2} - \frac{r}{2t} = \frac{t - r}{2t}$

31. $\tan\frac{R}{2} = \sqrt{\frac{1 - \cos R}{1 + \cos R}} = \sqrt{\frac{1 - \frac{s}{t}}{1 + \frac{s}{t}}} = \sqrt{\frac{t - s}{t + s} \cdot \frac{t + s}{t + s}}$
$= \sqrt{\frac{t^2 - s^2}{(t + s)^2}} = \sqrt{\frac{r^2}{(t + s)^2}} = \frac{r}{t + s}$

32. $\tan^2 \frac{S}{2} = (\tan \frac{S}{2})^2 = \left(\pm\sqrt{\frac{1 - \cos S}{1 + \cos S}}\right)^2 = \frac{1 - \cos S}{1 + \cos S}$

$= \dfrac{1 - \frac{r}{t}}{1 + \frac{r}{t}} = \frac{t - r}{t + r}$

33. No; since the sine function is periodic, A and B can have many different values.

34. $-\frac{24}{25}$

35. $-\frac{7}{25}$

36. $\frac{24}{7}$

37. $-\frac{25}{24}$

38. $\frac{\sqrt{5}}{5}$

39. $-\frac{2\sqrt{5}}{5}$

40. $-\frac{1}{2}$

41. -2

42. $\cos\theta\,(8\sin\theta - 3) = 0$
$\frac{\pi}{2}, \frac{3\pi}{2}$, 0.384, 2.757

43. $\sin\theta\,(4\cos\theta - 3) = 0$
$0, \pi$, 0.723, 5.560

44. $\cos\theta\,(2\sin^2\theta - 1) = 0$
$\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

45. $4\cos^2\theta - 1 = 0$
$\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

46. 1

47. $-\cos\theta$

48. $\cos\theta - \sin\theta$

49. Answers may vary. Sample:

a. $\sin 60° = \frac{\sqrt{3}}{2}$ $\cos 60° = \frac{1}{2}$

b. $\sin 120° = \frac{\sqrt{3}}{2}$

c. $\cos 30° = \frac{\sqrt{3}}{2}$

50. Answers may vary. Sample: No; the graphs of $y = \frac{\tan\theta}{4}$ and $y = \tan\frac{\theta}{4}$ intersect but do not coincide.

51–56. Answers may vary.

51. $4\sin\theta\cos\theta\,(\cos^2\theta - \sin^2\theta)$

52. $8\cos^4\theta - 8\cos^2\theta + 1$

53. $\dfrac{4\tan\theta\,(1 - \tan^2\theta)}{\tan^4\theta - 6\tan^2\theta + 1}$

54. $\pm\sqrt{\frac{1}{2} \mp \frac{1}{2}\sqrt{\frac{1}{2} + \frac{1}{2}\cos\theta}}$

55. $\pm\sqrt{\frac{1}{2} \pm \frac{1}{2}\sqrt{\frac{1}{2} + \frac{1}{2}\cos\theta}}$

56. $\pm\sqrt{\dfrac{1 \mp \sqrt{\frac{1}{2} + \frac{1}{2}\cos\theta}}{1 \pm \sqrt{\frac{1}{2} + \frac{1}{2}\cos\theta}}}$

57. a. $\tan \frac{A}{2} = \pm\sqrt{\frac{1 - \cos A}{1 + \cos A}} = \pm\sqrt{\frac{1 - \cos A}{1 + \cos A}} \cdot \sqrt{\frac{1 + \cos A}{1 + \cos A}} =$

$\pm\sqrt{\frac{1 - \cos^2 A}{(1 + \cos A)^2}} = \pm\sqrt{\frac{\sin^2 A}{(1 + \cos A)^2}} = \frac{\sin A}{1 + \cos A}$

Since $\tan \frac{A}{2}$ and $\sin A$ always have the same sign, only the positive sign occurs.

b. $\tan \frac{A}{2} = \pm\sqrt{\frac{1 - \cos A}{1 + \cos A}} = \pm\sqrt{\frac{1 - \cos A}{1 + \cos A}} \cdot \sqrt{\frac{1 - \cos A}{1 - \cos A}} =$

$\pm\sqrt{\frac{(1 - \cos A)^2}{1 - \cos^2 A}} = \pm\sqrt{\frac{(1 - \cos A)^2}{\sin^2 A}} = \frac{1 - \cos A}{\sin A}$

Since $\tan \frac{A}{2}$ and sin A always have the same sign, only the positive sign occurs.