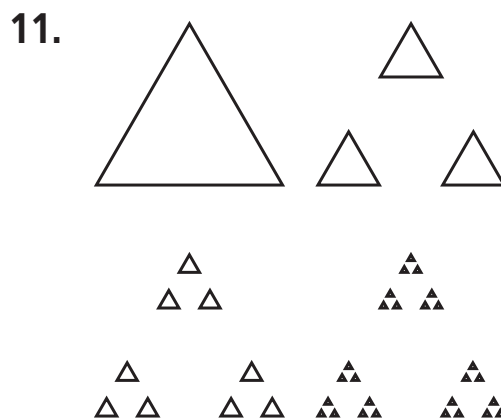


Answers for Lesson 11-1 Exercises

- Subtract 3; 65, 62, 59.
- Multiply by 2; 128, 256, 512.
- Add one more to each term (add 3, add 4, add 5, etc.); 25, 33, 42.
- Add 3; 16, 19, 22.
- Divide by 10; 0.001, 0.0001, 0.00001.
- Multiply by $\frac{1}{2}$; $\frac{1}{64}$, $\frac{1}{128}$, $\frac{1}{256}$.
- Multiply by -2 ; -128 , 256 , -512 .
- The n th term is the preceding term multiplied by n ; 720, 5040, 40,320.
- Every odd-numbered term is 0, and every even-numbered term is $\frac{1}{n-1}$; $0, \frac{1}{7}, 0$.



- $a_n = a_{n-1} + 1, a_1 = -2; 3$
- $a_n = a_{n-1} - 2, a_1 = 43; 33$
- $a_n = \frac{a_{n-1}}{2}, a_1 = 40; \frac{5}{4}$
- $a_n = a_{n-1} - 5, a_1 = 6; -14$
- $a_n = \frac{a_{n-1}}{4}, a_1 = 144; \frac{9}{16}$
- $a_n = a_{n-1} \cdot \frac{1}{2}, a_1 = \frac{1}{2}; \frac{1}{64}$
- $a_n = n + 3; 15$
- $a_n = \frac{1}{n+1}; \frac{1}{13}$
- $a_n = 3n + 1; 37$
- $a_n = 4n - 1; 47$
- $a_n = \frac{n-6}{2}; 3$
- $a_n = n^2 + 1; 145$

Answers for Lesson 11-1 Exercises (cont.)

24. recursive; 3, 9, 21, 45, 93
25. explicit; 0, 1, 3, 6, 10
26. explicit; $-24, -21, -16, -9, 0$
27. recursive; $-2, 6, -18, 54, -162$
28. explicit; $-6, -18, -38, -66, -102$
29. explicit; 3, 9, 19, 33, 51
30. explicit; 5, 10, 15, 20, 25
31. recursive; 340, 323, 306, 289, 272
32. 15, 26, 40
33. 20, 23; $a_n = 3n + 2$; explicit OR
 $a_n = a_{n-1} + 3, a_1 = 5$; recursive
34. 96, 192; $a_n = 3 \cdot 2^{n-1}$; explicit OR
 $a_n = 2a_{n-1}, a_1 = 3$; recursive
35. 216, 343; $a_n = n^3$; explicit
36. 4096, 16,384; $a_n = 4^n$; explicit OR
 $a_n = 4a_{n-1}; a_1 = 4$, recursive
37. 144, 169; $a_n = (n + 6)^2$; explicit OR
 $a_{n+1} = a_n + 2n + 13; a_1 = 49$, recursive
38. $-1, 1$; $a_n = -1(a_{n-1}), a_1 = -1$; recursive OR
 $a_n = (-1)^n$; explicit
39. $-1, -\frac{1}{2}$; $a_n = \frac{a_{n-1}}{2}, a_1 = -16$; recursive OR $a_n = \frac{-32}{2^n}$;
explicit
40. $-47, -40$; $a_n = a_{n-1} + 7, a_1 = -75$; recursive OR
 $a_n = -82 + 7_n$; explicit
41. $-11, -19$; $a_n = a_{n-1} - 8, a_1 = 21$; recursive OR
 $a_n = 29 - 8n$; explicit

Answers for Lesson 11-1 Exercises (cont.)

42. a_{n-2}, a_{n+2}

43. Answers may vary. Sample: A recursive formula requires that the previous term be known to find a given term. An explicit formula only requires the number of the term.

44. a-c. Answers may vary. Sample:

a. $1, -2, 4, -8, \dots$

b. $a_n = -2(a_{n-1}), a_1 = 1; a_n = (-2)^{n-1}$

c. $-524,288$

45. $28, 53, 89, 138$

46. $24, 78, 240, 726$

47. $25, 36, 49, 64$

48. $54, 128, 250, 432$

49. $\frac{16}{5}, \frac{25}{6}, \frac{36}{7}, \frac{49}{8}$

50. $\frac{5}{6}, \frac{6}{7}, \frac{7}{8}, \frac{8}{9}$

51. a. 25 boxes

b. 110 boxes

c. 9 levels

52. $a_n = 10 \cdot 2^{n-1}$

53. $a_n = -n - 4$

54. $a_n = -2 \cdot \left(\frac{1}{2}\right)^{n-1}$

55. $a_n = 1 + 4(n - 1)$

56. a. $a_n = a_{n-1} + 5, a_1 = 25; a_n = 20 + 5n$

b. \$40

c. $a_n = (a_{n-1} + \$20) \cdot 1.005, a_1 = \40.20

d. 6.5%

57. a. 15, 21

b. $a_n = a_{n-1} + n, a_1 = 1$

c. Yes; the formula yields the same values as the recursive formula.

Answers for Lesson 11-2 Exercises

1. no 2. yes; 10 3. no 4. no
5. yes; 3 6. yes; -11 7. yes; 4 8. no
9. no 10. no 11. 127 12. 0.3
13. 12.5 14. 0.0085 15. 225 16. -159
17. -59 18. 240 19. -146 20. 137
21. -7.5 22. 21 23. 13 24. 16
25. -7 26. 660 27. 7.5 28. 2.5
29. a_{11} or $\frac{a_{10} + a_{12}}{2}$ 30. 82.5 31. 4
32. $\frac{1}{2}$ 33. 13 34. 120 35. -19.5
36. 1.1 37. -1 38. $\frac{4}{5}$ 39. $\frac{r + s}{2}$
40. $\frac{2r + s}{2}$ 41. 0 42. $2x + 1$
43. The student assumed that the sequence was $a_n = 2^{n-1}$. However, $a_1 = 2^0 = 1$, not 0 as given in the problem.
44. a. Answers may vary. Sample: 25, 18, 11, 4, -3 , $-10, \dots$; to find the n th term, multiply $n - 1$ times (-7) and add to a_1 .
- b. Answers may vary. Sample: Start with the first term and continue to subtract 7 for each term. For each term, you subtract 7 times (term number -1) from the first term.
45. Answers may vary. Sample: An advantage of a recursive formula is that only the preceding term must be known to find the next term; a disadvantage is that many calculations may be required to find a term. An advantage of an explicit formula is that it is easy to find any term.
46. 23 47. 15 48. 18.5 49. 22
50. 6 51. 29

Answers for Lesson 11-2 Exercises (cont.)

52. $a_n = 2 + 2(n - 1); a_n = a_{n-1} + 2, a_1 = 2$

53. $a_n = 0 + 6(n - 1); a_n = a_{n-1} + 6, a_1 = 0$

54. $a_n = -5 + 1(n - 1); a_n = a_{n-1} + 1, a_1 = -5$

55. $a_n = -4 - 4(n - 1); a_n = a_{n-1} - 4, a_1 = -4$

56. $a_n = -2 + 7(n - 1); a_n = a_{n-1} + 7, a_1 = -2$

57. $a_n = 27 - 12(n - 1); a_n = a_{n-1} - 12, a_1 = 27$

58. $a_n = -5 + 1.5(n - 1); a_n = a_{n-1} + 1.5, a_1 = -5$

59. $a_n = -32 + 12(n - 1); a_n = a_{n-1} + 12, a_1 = -32$

60. $a_n = 1 + \frac{1}{3}(n - 1); a_n = a_{n-1} + \frac{1}{3}, a_1 = 1$

61. $a_n = \frac{1}{8}(n - 1); a_n = a_{n-1} + \frac{1}{8}, a_1 = 0$

62. 6 min; 1 min

63. -4, -10, -16

64. 4.6, -0.8, -6.2

65. -8, -17, -26

66. $\frac{19}{5}, 5, \frac{31}{5}$

67. 17, 17, 17

68. 681, 702, 723

69. -12.5, -8, -3.5

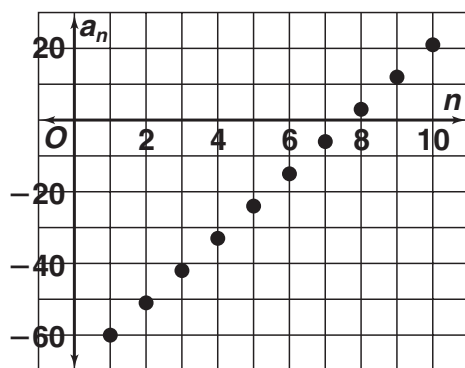
70. $a + 5, a + 9, a + 13$

71. a. \$20, \$45, \$75, \$110, \$150, \$195, \$245, \$300, \$360, \$425, \$495

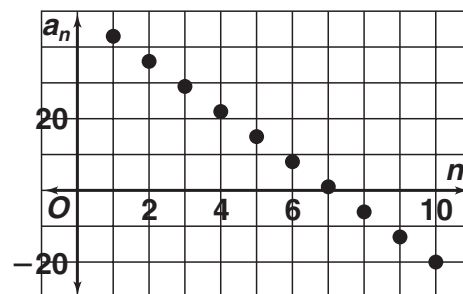
b. $a_n = a_{n-1} + \$20 + \$5(n - 1), a_1 = \$20$

c. \$495

72.



73.



Answers for Lesson 11-2 Exercises (cont.)

74. 2.46 min or 2 min 28 s

75. 54

76. 8

77. 21st term

78. 43rd term

79. $a_1 = -1, d = 3$

80. $a_1 = -4, d = 4$

81. $a_1 = 52, d = -10$

82. $a_1 = -21\frac{1}{4}, d = 4\frac{1}{4}$

83. $a_1 = -100.5, d = 22$

84. $a_1 = -9, d = 2.2$

85. $9k + 32$

86. $21k - 43$

Answers for Lesson 11-3 Exercises

1. yes; 2; 16, 32
2. no
3. yes; -2 ; 16, -32
4. yes; -1 ; -1 , 1
5. yes; 0.4; 0.256, 0.1024
6. yes; 0.1; 0.0007, 0.00007
7. yes; $-\frac{1}{3}$; $\frac{2}{9}$, $-\frac{2}{27}$
8. no
9. yes; 1.5; 50.625, 75.9375
10. yes; -5 ; 1250, -6250
11. yes; 6; -1296 , -7776
12. no
13. $a_n = 5 \cdot (-3)^{n-1}$; 5, -15 , 45, -135 , 405
14. $a_n = 0.0237 \cdot 10^{n-1}$; 0.0237, 0.237, 2.37, 23.7, 237
15. $a_n = \frac{1}{2} \left(\frac{2}{3}\right)^{n-1}$; $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{9}$, $\frac{4}{27}$, $\frac{8}{81}$
16. $a_n = 0.5^{n-1}$; 1, 0.5, 0.25, 0.125, 0.0625
17. $a_n = 100(-20)^{n-1}$; 100, -2000 , 40,000, $-800,000$, 16,000,000
18. $a_n = 7 \cdot 1^{n-1}$; 7, 7, 7, 7, 7
19. $a_n = 1024(0.5)^{n-1}$; 1024, 512, 256, 128, 64
20. $a_n = 4(0.1)^{n-1}$; 4, 0.4, 0.04, 0.004, 0.0004
21. $a_n = 10(-1)^{n-1}$; 10, -10 , 10, -10 , 10
22. 67.5 or -67.5
23. 1530 or -1530
24. $\frac{4}{15}$ or $-\frac{4}{15}$
25. 1.5 or -1.5
26. 3.75 or -3.75
27. 6 or -6
28. geometric; 720, 1440
29. arithmetic; 125, 150
30. geometric; 3, -3
31. arithmetic; 50, 55
32. geometric; -80 , 160
33. geometric; 0.125, 0.0625
34. neither; 20, 26
35. either; 2, 2
36. neither; 25, 36
37. 6561, 2187, 729, or -6561 ; 2187, -729

Answers for Lesson 11-3 Exercises (cont.)

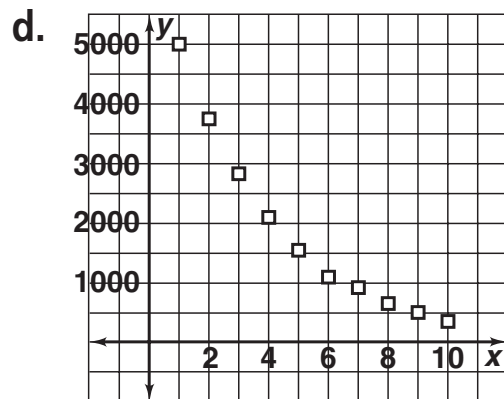
38. 7.5, 22.5, 67.5, or $-7.5, 22.5, -67.5$
39. 10, 8, 6.4, or $-10, 8, -6.4$
40. $-6.64, -11.02, -18.30$, or $6.64, -11.02, 18.30$
41. a–d. Answers may vary. Sample:
- a. 3 and 12; 6
 - b. 3, 6, 12; 2
 - c. 768
 - d. 48; 5th term
42. 768
43. 12,288
44. 786,432
45. 201,326,592
46. 3×4^{16} or 12,884,901,888
47. $3(4^{n-1})$
48. 4
49. 16
50. 2.5
51. 10
52. $-\frac{1}{6}$
53. $-\frac{2}{3}$
54. Both the common difference and the common ratio are used to find the next term in a sequence, but a common difference is added and a common ratio is multiplied.
55. \$142.79, \$613.59, \$28.62, \$58.92, \$105.82, \$262.94
56. a. $d, \frac{1}{2}d, \frac{1}{4}d, \frac{1}{8}d, \frac{1}{16}d, \frac{1}{32}d$
- b. Yes; the common ratio is $\frac{1}{2}$.
- c. $a_n = a_{n-1} \cdot \frac{1}{2}, a_1 = d$
57. Both arithmetic and geometric sequence explicit formulas use the first term $a_1, n - 1$, and a common term. The recursive formulas both use a_{n-1} and a common term.

Answers for Lesson 11-3 Exercises (cont.)

58. a. 5000, 3750, 2812.5, 2109.38, 1582.03

b. $\frac{3}{4}$

c. 375.42 cm^3



e. The common ratio $\frac{3}{4}$ is less than one, so the graph is decreasing.

59. 7

60. 128

Answers for Lesson 11-4 Exercises

1. $21 + 18 + 15 + 12 + 9 + 6 + 3$; 84
2. $(-5) + (-15) + (-25) + (-35) + (-45)$; -125
3. $100 + 99 + 98 + 97 + 96 + 95$; 585
4. $0.5 + 0.25 + 0 + (-0.25) + (-0.5) + (-0.75)$; -0.75
5. $17.3 + 19.6 + 21.9 + 24.2 + 26.5$; 109.5
6. $4.5 + 5.6 + 6.7 + 7.8 + 8.9 + 10 + 11.1$; 54.6
7. 32
8. -48
9. 264
10. 35
11. 4292
12. -146
13. $\sum_{n=1}^4 2n$
14. $\sum_{n=1}^8 (n + 7)$
15. $\sum_{n=1}^7 (n + 4)$
16. $\sum_{n=1}^{11} (3n - 2)$
17. $\sum_{n=1}^{15} 7n$
18. $\sum_{n=1}^5 -3n$
19. 5, 1, 9; 25
20. 5, -3, -11; -35
21. 6, 4, -1; 9
22. 5, 0, 0.8; 2
23. $9, \frac{8}{3}, \frac{40}{3}$; 72
24. 6, 15, 10; 75
25. sequence; infinite
26. sequence; finite
27. series; finite
28. series; infinite
29. sequence; infinite
30. series; finite
31. a. 270 chairs on each side, 390 chairs middle, 930 chairs total
 b. each side: $\sum_{n=1}^{20} (n + 3)$; middle: $\sum_{n=1}^{20} (n + 9)$
 c. \$46,950
32. a. 8; the formula for the corresponding sequence is $a_n = 3n + 7$. Solving $3n + 7 = 31$ for n shows that $n = 8$.
 b. 164

Answers for Lesson 11-4 Exercises (cont.)

33. a. 91

b. 83

34. a. $a_n = n + 1$

b. $\sum_{n=1}^9 (n + 1)$

c. 18 cans

d. No; no; 13 rows have 104 cans, 14 rows have 119 cans, 15 rows have 135 cans, and 16 rows have 152 cans. The number of rows would not be an integer for 110 cans or 140 cans.

35. 110

36. -765

37. 5150

38. -22

39. -0.6

40. 1,000,500

41. a. No; $75 + 25(6) = 225$, which is less than 500.

b. Answers may vary. Sample: Pro: spreading the cost over several years, con: calculators purchased first may be outdated by the time 500 calculators have been purchased; check students' work.

42. a–d. Check students' work.

43. 300

44. -200

45. 34

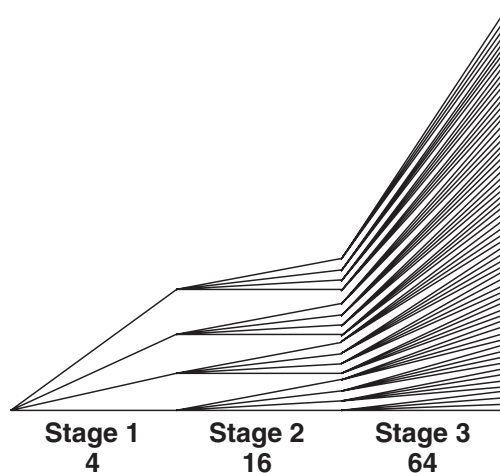
46. 37

47. $10x + 45y$

48. $45x - 210y$

Answers for Lesson 11-5 Exercises

1. 255
2. 1456
3. 381
4. 3647
5. $-10,235$
6. $-\frac{1111}{6}$
7. $\frac{255}{256}$
8. -1640
9. converges; has a sum
10. converges; has a sum
11. converges; has a sum
12. diverges; no sum
13. diverges; no sum
14. converges; has a sum
15. diverges; no sum
16. converges; has a sum
17. diverges; no sum
18. $1.\bar{2}$
19. 1
20. $\frac{5}{6}$
21. $\frac{9}{2}$
22. 9
23. $\frac{9}{5}$
24. geometric; 2046
25. arithmetic; 420
26. geometric; $-1,627,605$
27. geometric; ≈ 96.47
28. arithmetic; 500,500
29. geometric; ≈ 121.5
30. a.



4, 16, 64

b. $4 + 16 + 64 + 256 + 1024 + 4096$

c. 5460

Answers for Lesson 11-5 Exercises (cont.)

31. a. 20, 18, 16.2, 14.58

b. ≈ 198.59

c. $S = \frac{20}{1 - 0.9} = 200$

d. Check students' work.

32. $\frac{5}{4}$

33. 4

34. $\frac{3}{4}$

35. no sum

36. $0.8\bar{3}$

37. no sum

38. a. 2

b. $25 + 20 + 16 + 12.8 + 10.24 + \dots$

c. $20 + 16 + 12.8 + 10.24 + \dots$

d. 225 cm

39. Check students' work.

40. a. No; the sum of a series of positive numbers will be positive.

b. Your classmate did not check to see if $|r|$ was less than 1.

41. $\frac{7}{8}$

42. 10

43. (b); (a) yields \$26,000; using the formula for finding the sum of a finite geometric series, (b) yields \$1,342,177.26.

44. a. Answers may vary. Sample: The student used $r - 1$ instead of $1 - r$ in the formula for the sum of an infinite geometric series.

b. $\frac{1}{2}$

45. converges; 1

46. diverges

47. converges; ≈ 2.718

48. a. 70th swing

b. 10,000 cm

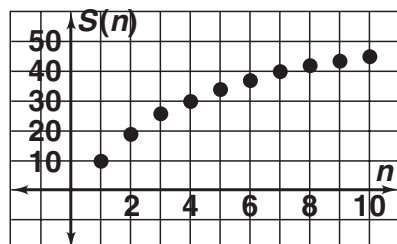
Answers for Lesson 11-5 Exercises (cont.)

49. a. $S = \frac{0.142857}{1 - 0.000001} = \frac{0.142857}{0.999999} = \frac{1}{7}$

b. $\frac{3}{7}$

50. a. all integers greater than or equal to 1

b. 10; 18; 24.4; 29.52; 33.62; 36.89; 39.51; 41.61; 43.29; 44.63;

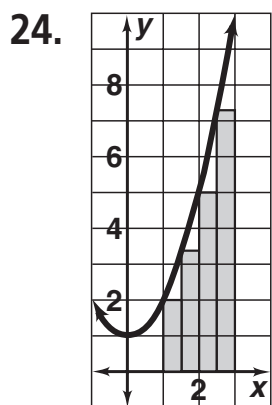


c. 50

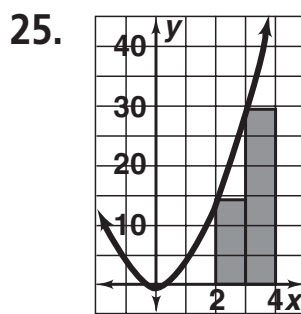
Answers for Lesson 11-6 Exercises

- | | | |
|---------------------------------------|-------------------------------------|-----------------------------|
| 1. total produced | 2. amount of growth | |
| 3. miles | 4. distance traveled | |
| 5. total price | 6. ≈ 270 units ² | |
| 7. 100 units ² | 8. 480 units ² | |
| 9. $A = \sum_{n=1}^2 1f(a_n)$ | 10. $A = \sum_{n=1}^2 1f(a_n)$ | |
| a. 0.5 units ² | a. 5 units ² | |
| b. 2.5 units ² | b. 9 units ² | |
| 11. $A = \sum_{n=1}^2 1g(a_n)$ | 12. $A = \sum_{n=1}^2 1f(a_n)$ | |
| a. 3 units ² | a. 3 unit ² | |
| b. 7 units ² | b. 7 units ² | |
| 13. $A = \sum_{n=1}^2 1f(a_n)$ | 14. $A = \sum_{n=1}^2 1h(a_n)$ | |
| a. $10\frac{2}{3}$ units ² | a. 5 units ² | |
| b. $13\frac{1}{3}$ units ² | b. 25 units ² | |
| 15. $A = \sum_{n=1}^2 1f(a_n)$ | 16. $A = \sum_{n=1}^2 1h(a_n)$ | |
| a. 6.75 units ² | a. 5 units ² | |
| b. 7.75 units ² | b. 9 units ² | |
| 17. $A = \sum_{n=1}^2 1f(a_n)$ | | |
| a. 5 units ² | | |
| b. 9 units ² | | |
| 18. $1\bar{6}$ units ² | 19. 2.5 units ² | 20. 0.25 units ² |
| 21. 3.3 units ² | 22. $2\bar{3}$ units ² | 23. 3.25 units ² |

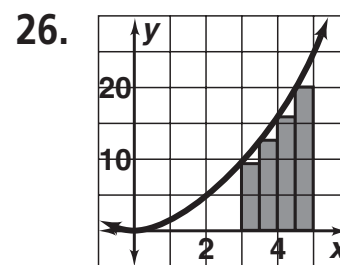
Answers for Lesson 11-6 Exercises (cont.)



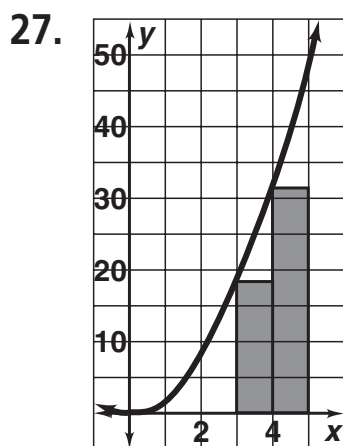
8.75 units²



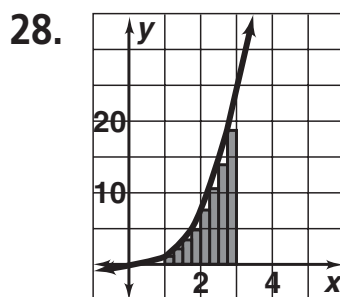
43 units²



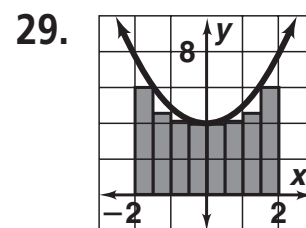
28.75 units²



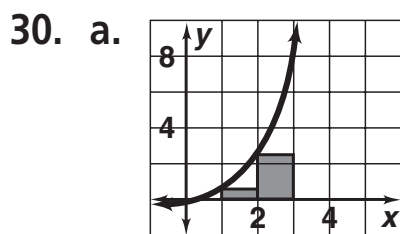
50 units²



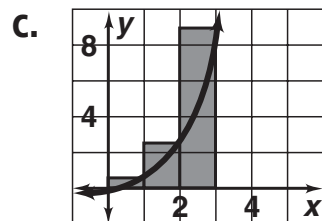
16.875 units²



19.5 units²



b. 3 units²

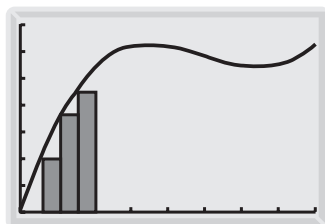


12 units²

d. 7.5 units²; the mean best approximates the area because it is between the other measures known to be larger and smaller than the actual value.

Answers for Lesson 11-6 Exercises (cont.)

31. a. Answers may vary. Sample:



Xmin:0 Ymin:0
 Xmax:200 Ymax:70
 Xscl:25 Yscl:10

≈ 0.37 miles

b. Answers may vary. Sample: Using inscribed rectangles will result in an estimate smaller than the actual number.

32. 9 units^2

33. 7.5 units^2

34. 15 units^2

35. $\approx 3.46 \text{ units}^2$

36. $\approx 8.05 \text{ units}^2$

37. 9.75 units^2

38. 10 units^2

39. 2.5 units^2

40. 9.9 units^2

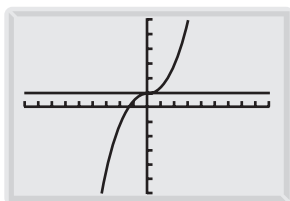
41. Check students' work.

42. a. 17.5 units^2

b. 14 units^2

c. The estimate in (a) is closer to the actual area because there is less area between the curve and the rectangles when more rectangles are used.

43. a.

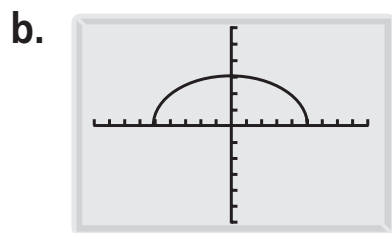


b. The area under both curves is the same, 3 units^2 , over the interval $-1.5 \leq x \leq 1.5$; this is true because the amount of area *above* $y = \frac{1}{4}x^3 + 1$ and *below* $y = 1$ on the left side of the y -axis is equal to the area *below* $y = \frac{1}{4}x^3 + 1$ and *above* $y = 1$ on the right side of the y -axis.

Answers for Lesson 11-6 Exercises (cont.)

44. 6 units^2

45. a. $y = \sqrt{9 - \frac{9x^2}{25}}$



$\approx 23.56 \text{ units}^2$

c. $\approx 47.12 \text{ units}^2$

d. Check students' work.