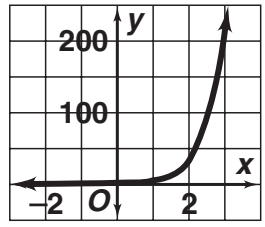
Answers for Lesson 8-1 Exercises

1.



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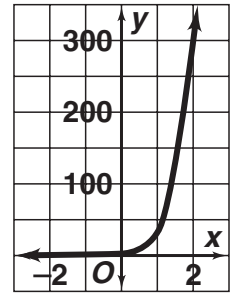

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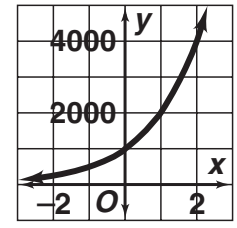

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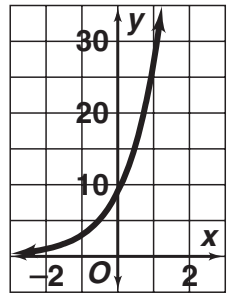

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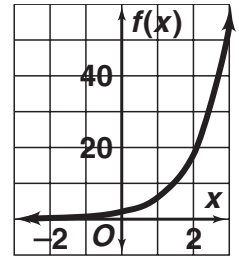

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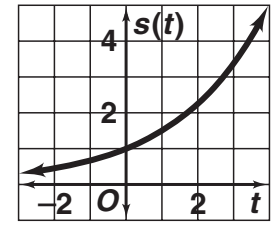

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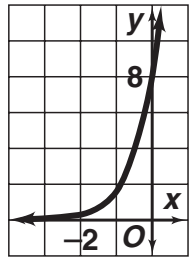

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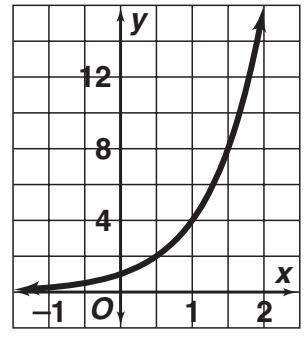


9. a. 1.0126

b. $y = 6.08(1.0126)^x$, where $x = 0$ corresponds to 2000

10. $y = 0.5(2)^x$

11. $y = 2.5(7)^x$

12. $y = 8(1.5)^x$

13. $y = 5(0.6)^x$

14. $y = 3(0.5)^x$

15. $y = 24\left(\frac{1}{3}\right)^x$

16. exponential growth

17. exponential decay

18. exponential growth

19. exponential decay

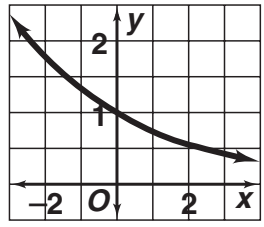
20. exponential decay

21. exponential growth

22. exponential growth

23. exponential decay

24.



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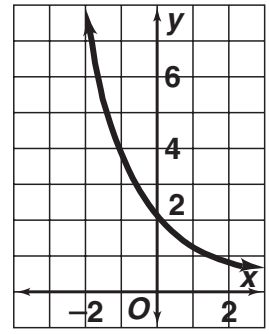

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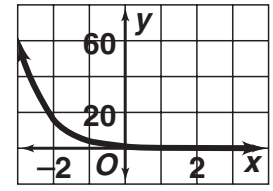

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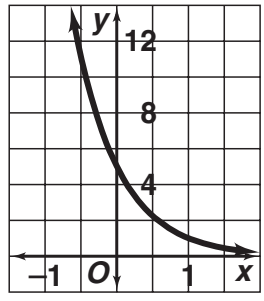

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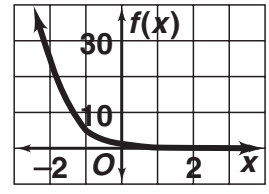

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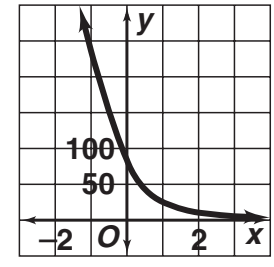

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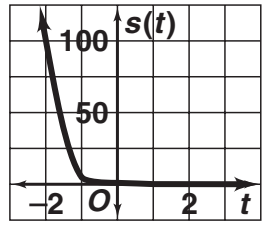

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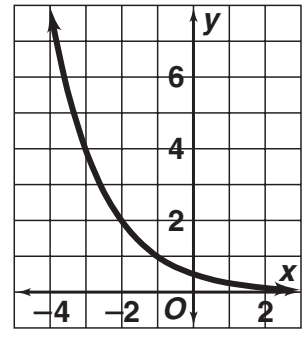


32. $y = 100(0.5)^x$; 1.5625

33. $y = 12{,}000(0.9)^x$; 6377

34. $y = 12{,}000(0.1)^x$; 0.012

35. **a.** $y = 6500(0.857)^x$

b. \$4091.25

36. 63% increase

37. 30% increase

38. 35% decrease

39. 70% increase

40. 87.5% decrease

41. 75% decrease

42. D

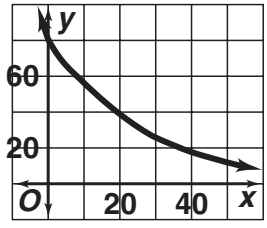
43. **a.** about 5.6%

b. about 0.0017%

44. **a.** Answers may vary. Sample: $y = 30{,}000(0.7)^x$ for car 1 $y = 15{,}000(0.8)^x$ for car 2; car 2 will be worth more.

b. Check students' work.

45. **a.** $y = 80(0.965)^x$

b. 

about 47 years

46. 1.70 **47.** 6 **48.** 0.25 **49.** 0.45

50. 1.125 **51.** 0.999 **52.** 1.001 **53.** 2

54. $y = 34(1.22)^x$, where x represents the number of years since 1995.

55. Check students' work.

56. about \$42,140

57. C

58. B; the graph shows a decreasing function, so $b < 1$, which eliminates A. The y = values are all positive, which eliminates C.

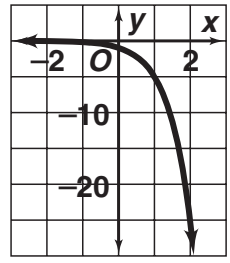
59. **a.** A negative growth rate would be represented by adding the negative rate to 1.

b. Armenia: $y = 11.8(1.099)^x$, Canada: $y = 958.7(1.017)^x$, Germany: $y = 2271(0.999)^x$, Venezuela: $y = 117.9(0.908)^x$. x in each equation represents the number of years since 2003.

c. Armenia: \$18.9 billion, Canada: \$1043 billion, Germany: \$2260 billion, Venezuela: \$72.8 billion

1-8. Asymptote is $y = 0$.

1.



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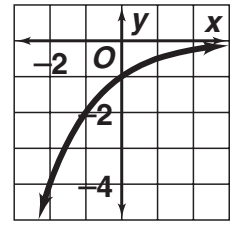

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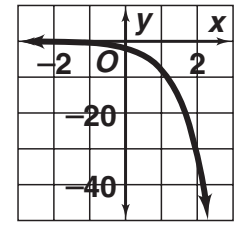

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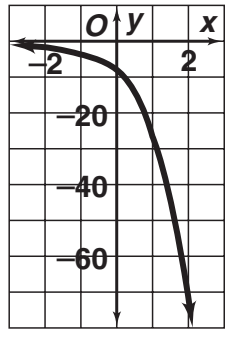

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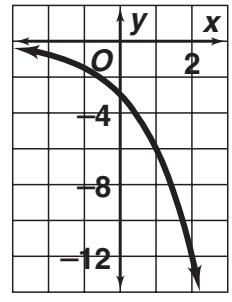

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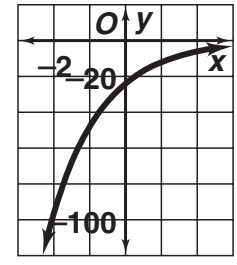

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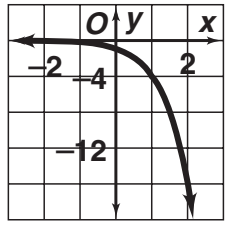

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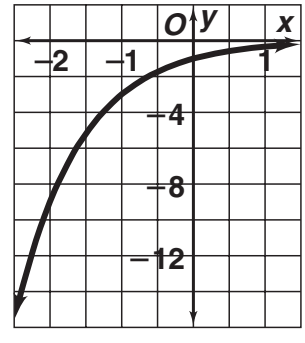

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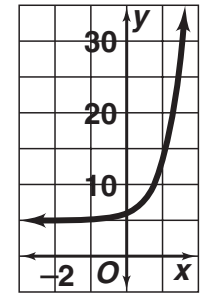

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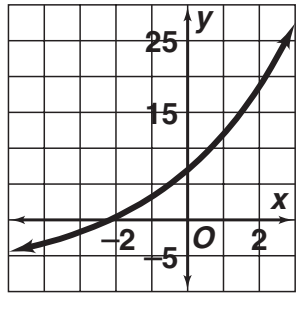

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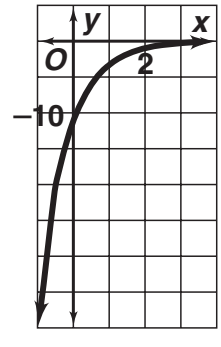

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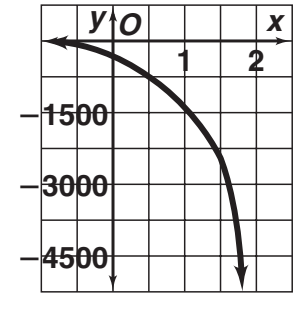

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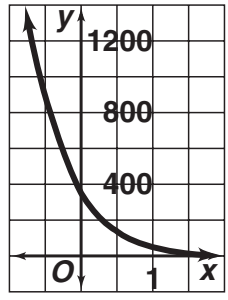

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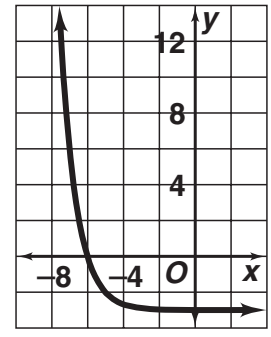


15. $y = 50\left(\frac{1}{2}\right)^{\frac{1}{14.3}x}$; 0.85 mg

16. $y = 200\left(\frac{1}{2}\right)^{\frac{1}{8.14}x}$; 0.43 mg

17. $y = 24\left(\frac{1}{2}\right)^{\frac{1}{5730}x}$; 0.64 mg

18. 20.0855

19. 403.4288

20. 0.1353

21. 1

22. 12.1825

23. 15.1543

24. \$2330.65

25. \$448.30

26. \$1819.76

27. 0

28. 1

29. If $c < 0$, the graph models exponential decay. If $c = 0$, the graph is a horizontal line. If $c > 0$, the graph models exponential growth.

30. \$6168.41

31. **a.** Answers may vary. Sample: $y = -2(1.3)^x$

b. Answers may vary. Sample: I am in debt for \$2 and my debt is growing at a rate of 30% per year.

c. The graph of exponential decay approaches the asymptote $y = 0$ as x increases. The graph of negative exponential growth approaches the asymptote $y = 0$ as x decreases.

32. $y = 4\left(\frac{1}{2}\right)^x$; $y = 4\left(\frac{1}{2}\right)^{x+4} + 3$

33. $y = -3^x$; $y = -3^{x-8} + 2$

34. $y = \frac{1}{2}(2)^x$; $y = \frac{1}{2}(2)^{x-6} - 7$

35. $y = -3\left(\frac{1}{3}\right)^x$; $y = -3\left(\frac{1}{3}\right)^{x+5} - 1$

36. 75.0 pascals

37. 8.7 yr

38. A deficit that is growing exponentially is modeled by $y = ab^{cx}$, where $a < 0$, and either $b > 1$ and $c > 0$ or $0 < b < 1$ and $c < 0$.

39. B

40. **a.** \$2501.50

b. \$3.15 more

41. \$399.97

42. exponential growth

43. exponential growth

44. exponential decay

45. exponential growth

46. exponential decay

47. exponential growth

48. **a.** $y = 8001 - 3^x$, where y is the number of uninfected people and x represents days.

b. 5814 people

c. about 9 days

49. **a.** about 10 names; about 24 names

b. Graphically, it will never happen; the graph has $y = 30$ as an asymptote. (In reality, you would be close to knowing all the names in about 21 days.)

c. Answers may vary. Sample: I learn names pretty quickly; my learning rate might be 0.4.

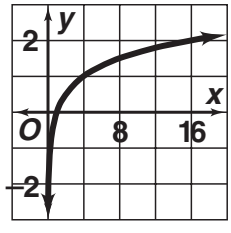
50. **a.** 2928 m^3

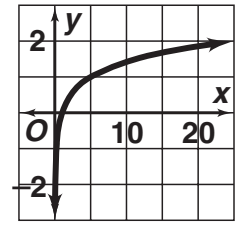
b. $V = 2928 - 15(2^x - 1)$

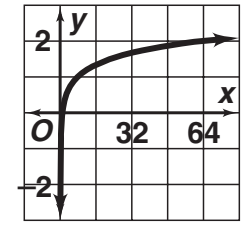
c. ninth weekend

Answers for Lesson 8-3 Exercises

1. The earthquake in Missouri released about 1.97 times more energy.
2. The earthquake in Chile released about 231 times more energy.
3. The earthquake in Missouri released about 8,759,310 times more energy.
4. The earthquake in Missouri released about 30 times more energy.
5. The earthquake in Alaska released about 83 times more energy.

6. $\log_7 49 = 2$
7. $3 = \log 1000$
8. $\log_5 625 = 4$
9. $\log \frac{1}{10} = -1$
10. $2 = \log_8 64$
11. $\log_{\frac{1}{2}} 4 = -2$
12. $3 = \log_{\frac{1}{3}}\left(\frac{1}{27}\right)$
13. $-2 = \log 0.01$
14. 4
15. $\frac{1}{2}$
16. 1
17. $\frac{3}{2}$
18. 3
19. $\frac{1}{2}$
20. undefined
21. 2
22. 5
23. 1
24. 4
25. 3
26. 6.3×10^{-6}
27. 6.3×10^{-3}
28. 1.0×10^{-8}
29. 7.9×10^{-4}
30. 5.0×10^{-7}
31. 1.3×10^{-5}
32. 1.0×10^{-4}
33. 4.0×10^{-6}
34. 2.5×10^{-4}
35. 

36. 

37. 

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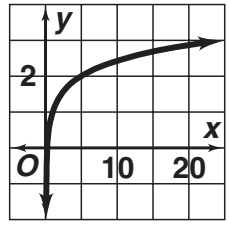

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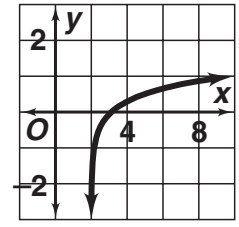

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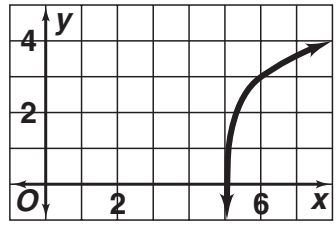


41. 0.6990; 0

42. $-4.2147; -5$

43. $-1.0969; -2$

44. 2.3010; 2

45. $-0.7782; -1$

46. 1.2435; 1

47. 7.1139; 7

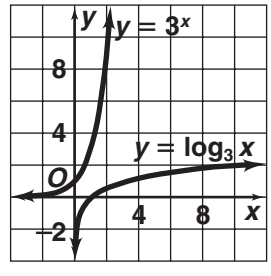
48. 0.5119; 0

49. apple juice: 3.5, acidic; buttermilk: 4.6, acidic; cream: 6.6, acidic; ketchup: 3.9, acidic; shrimp sauce: 7.1, basic; strained peas: 6, acidic

50. The error is in the second line. It should read $3 = 27^x$; the correct answer is $\frac{1}{3}$.

51. First rewrite $y = \log_1 x$ as $1^y = x$. For any real number y, $x = 1$.

52. Answers may vary. Sample: $y = \log_3 x$; $y = 3^x$



53. $128 = 2^7$

54. $0.0001 = 10^{-4}$

55. $16{,}807 = 7^5$

56. $6 = 6^1$

57. $1 = 4^0$

58. $\frac{1}{9} = 3^{-2}$

59. $\frac{1}{2} = 2^{-1}$

60. $10 = 10^1$

61. $8192 = 2^{13}$

62. **a.** buffalo bone: 9826 to 10,128 years old,
bone fragment: 9776 to 10,180 years old,
pottery shard: 0 to 183 years old,
charcoal: 9718 to 10,242 years old,
spear shaft: 9776 to 10,263 years old

b. The pottery shard; answers may vary. Samples: the pottery may be from a later civilization, or the mass or the beta radiation emissions may have been measured incorrectly.

63. C

64. $y = 4^x$

65. $y = 0.5^x$

66. $y = 10^x$

67. $y = 2^{x-1}$

68. $y = 10^x - 1$

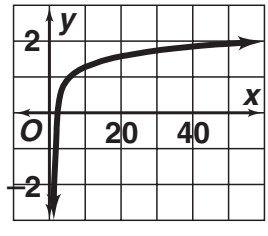
69. $y = 10^{x-1}$

70. $y = 10^x + 2$

71. $y = 5^{\frac{x}{2}}$

72. $y = a^x + b$

73.



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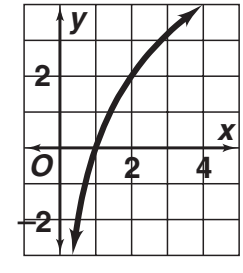
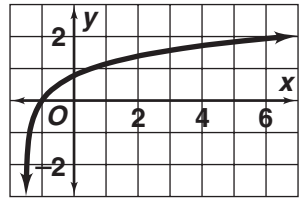


75. 

76. domain $\{x \mid x > 0\}$, range: all reals

77. domain $\{x \mid x > 0\}$, range: all reals

78. domain $\{x \mid x > 3\}$, range: all reals

79. domain $\{x \mid x > 0\}$, range: all reals

80. domain $\{x \mid x > 2\}$, range: all reals

81. domain $\{x \mid x > -1\}$, range: all reals

82. domain $\{x \mid x > 0\}$, range: all reals

83. domain $\{x \mid x > 0\}$, range: all reals

84. domain $\{x \mid x > t\}$, range: all reals

85. 100 **86.** 70 **87.** 60

88. 20 **89.** 10

90. **a.** II
b. III
c. I

Answers for Lesson 8-4 Exercises

1. Product Property

2. Quotient Property

3. Power Property

4. Power Property

5. Power Property, Quotient Property

6. Power Property

7. Power Property, Quotient Property

8. Power Property, Product Property

9. Power Property, Quotient Property

10. Power Property, Product Property

11. $\log 14$ **12.** $\log_2 3$ **13.** $\log 972$ **14.** $\log \frac{2}{3}$

15. $\log \frac{m^4}{n}$ **16.** $\log \frac{5}{2^k}$ **17.** $\log_6 5x$ **18.** $\log_7 \frac{xy}{z}$

19. $3 \log x + 5 \log y$

20. $\log_7 22 + \log_7 x + \log_7 y + \log_7 z$

21. $\log_4 5 + \frac{1}{2} \log_4 x$ **22.** $\log 3 + 4 \log m - 2 \log n$

23. $\log_5 r - \log_5 s$ **24.** $2 \log_3 2 + 2 \log_3 x$

25. $\log_3 7 + 2 \log (2x - 3)$ **26.** $2 \log a + 3 \log b - 4 \log c$

27. $\frac{1}{2} \log 2 + \frac{1}{2} \log x - \frac{1}{2} \log y$ **28.** $1 + \frac{1}{2} \log_8 3 + \frac{5}{2} \log_8 a$

29. $\log s + \frac{1}{2} \log 7 - 2 \log t$ **30.** $-\log_b x$

31. 9 dB **32.** 13 dB **33.** -2 **34.** 1

35. 6 **36.** 2 **37.** 2 **38.** 1

39. 1 **40.** -2 **41.** 1

42. The coefficient $\frac{1}{2}$ is missing in $\log_4 s$; $\log_4 \sqrt{\frac{t}{s}} = \frac{1}{2}\log_4 \frac{t}{s} = \frac{1}{2}(\log_4 t - \log_4 s) = \frac{1}{2}\log_4 t - \frac{1}{2}\log_4 s$.

43. Answers may vary. Sample: $\log 150 = \log 15 + \log 10$

44. 1.3803 **45.** 1.4772 **46.** 1.2042

47. 2.097 **48.** 0.1761 **49.** −0.0969

50. −0.6021 **51.** −1.398 **52.** 1.398

53. −0.7782 **54.** 1.5564 **55.** 0.3495

56. 12 dB **57.** 0.00001

58. True; $\log_2 4 = 2$ and $\log_2 8 = 3$.

59. False; $\frac{1}{2}\log_3 3 = \log_3 3^{\frac{1}{2}}$, not $\log_3 \frac{3}{2}$.

60. True; it is an example of the Power Property since $8 = 2^3$.

61. False; the two logs have different bases.

62. False; this is not an example of the Quotient Property. $\log (x - 2) \neq \log x - \log 2$.

63. False; $\log_b \frac{x}{y} = \log_b x - \log_b y$.

64. False; the exponent on the left means the log x, quantity squared, not the log of x^2.

65. False; $\log_4 7 - \log_4 3 = \log_4 \frac{7}{3}$, not $\log_4 4$.

66. True; $\log x + \log (x^2 + 2) = \log x(x^2 + 2)$, which equals $\log (x^3 + 2x)$.

67. False; the three logs have different bases.

68. True; the power and quotient properties are used correctly.

69. True; the left side equals $\log_b \left(\frac{1}{8} \cdot 4^3\right)$, which equals $\log_b 8$.

70. 102 dB

71. No; the expression $(2x + 1)$ is a sum, so it is not covered by the Product, Quotient, or Power properties.

72. The log of a product is equal to the sum of the logs. $\log (MN) = \log M + \log N$. So $\log (5 \cdot 2) = \log 10 = 1$, $\log 5 \cdot \log 2 \approx (0.7)(0.3) = 0.21$, which is not equal to 1.

73. $\log_3 \sqrt[4]{2x}$

74. $\log_x \frac{2\sqrt{y}}{z^3}$

75. $\log \frac{27}{2}$

76. $\log_4 \frac{m^x n^{1/y}}{p}$

77. $\log_b \frac{\sqrt[3]{x^2}\sqrt[4]{y^3}}{z^5}$

78. $\log \frac{\sqrt[4]{z}}{\sqrt[4]{3}\sqrt{x^5}}$

79. $3 \log 2 + \frac{3}{2} \log x - 3 \log 5$

80. $3 \log m - 4 \log n + 2 \log p$

81. $\log 2 + \frac{1}{2} \log 4 + \frac{1}{2} \log r - \log s$

82. $\frac{1}{2} \log_b x + \frac{2}{3} \log_b y - \frac{2}{5} \log_b z$

83. $\frac{5}{2} \log_4 x + \frac{7}{2} \log_4 y - \log_4 z - 4 \log_4 w$

84. $\frac{1}{2} \log (x^2 - 4) - 2 \log (x + 3)$

85. $\frac{1}{2} \log x + \frac{1}{4} \log 2 - \log y$

86. $\log_3 x + \log_3 y - 6 \log_3 z$

87. $\frac{1}{2} \log_7 (r + 9) - 2 \log_7 s - \frac{1}{3} \log_7 t$

88. $v = \log_b N$
$b^v = N$
$MN = b^u \cdot b^v = b^{u+v}$
$\log_b MN = u + v$
$\log_b MN = \log_b M + \log_b N$

89. **1.** $u = \log_b M$ (Given)

2. $b^u = M$ (Rewrite in exponential form.)

3. $(b^u)^x = M^x$ (Raise each side to x power.)

4. $b^{ux} = M^x$ (Power Property of exponents)

5. $\log_b b^{ux} = \log_b M^x$ (Take the log of each side.)

6. $ux = \log_b M^x$ (Simplify.)

7. $\log_b M^x = x \cdot \log_b M$ (Substitution)

90. **1.** $u = \log_b M$ (Given)

2. $b^u = M$ (Rewrite in exponential form.)

3. $v = \log_b N$ (Given)

4. $b^v = N$ (Rewrite in exponential form.)

5. $\frac{M}{N} = \frac{b^u}{b^v} = b^{u-v}$ (Quotient Property of Exponents)

6. $\log_b \frac{M}{N} = \log_b b^{u-v}$ (Take the log of each side.)

7. $\log_b \frac{M}{N} = u - v$ (Simplify.)

8. $\log_b \frac{M}{N} = \log_b M - \log_b N$ (Substitution)

Answers for Lesson 8-5 Exercises

1. 1.5850 **2.** 2.1240 **3.** 2.7320 **4.** 3.0101

5. 3 **6.** 3.4650 **7.** 0.9534 **8.** 1.330

9. 0.3579 **10.** 3.2056 **11.** 0.2720 **12.** 2.1073

13. 0.5690 **14.** 1.2871 **15.** 4.7027 **16.** 14.4894

17. 6 **18.** 4.89 **19.** 0.64 **20.** 009, 5.86

21. -0.73 **22.** 2.41

23. about 7.1 years **24.** about 2018

25. 3.1699; $\log_8 729$ **26.** 1.5; $\log_8 22.627$

27. 3.6309; $\log_8 1901.3$ **28.** 2.5643; $\log_8 206.93$

29. 3.1827; $\log_8 748.56$ **30.** 2.8074; $\log_8 343$

31. 3.8737; $\log_8 3149.6$ **32.** 0.0792; $\log_8 1.1790$

33. 0.05 **34.** $\frac{\sqrt{10}}{10}$, or ≈ 0.3162

35. 33 **36.** 10,000

37. $\frac{1}{60}$, or ≈ 0.0167 **38.** 12

39. $\sqrt{10}$, or ≈ 3.1623 **40.** $100\sqrt{10} - 1$, or ≈ 315.2

41. 2 **42.** 3×10^8

43. $100{,}000\sqrt{5}$, or $\approx 223{,}606.8$ **44.** 5

45. $\frac{1}{4}$ **46.** 1357.2

47. 7

48. **a.** 18.9658

b. 18.9658

c. Answers may vary. Sample: You don't have to use the Change of Base formula with the base-10 method, but there is less algebra with the base-2 method.

49. 5.1

50. $20 = 8(1.2)^x$, 5 years

51. $2 = 10\left(\frac{1}{2}\right)^{\frac{x}{1.17}}$, about 2.7 min

52. A

53. -1

54. 3

55. $\frac{1}{2}$

56. 3

57. $\frac{1}{3}$

58. -2

59. 3

60. $-\frac{1}{2}$

61. **a.** Let x equal the number of years after 2000. Florida growth factor $= 1.0213$, $y = 15{,}982{,}378\ (1.0213)^x$; New York growth factor $= 1.0054$, $y = 18{,}976{,}457 \cdot (1.0054)^x$

b. 2011

62. **a.** Texas growth factor $= 1.0208$, $y = 20{,}851{,}820\ (1.0208)^x$; California growth factor $= 1.013$, $y = 33{,}871{,}648 \cdot (1.013)^x$

b. 2063

63. Since Florida's growth rate is larger than Texas's growth rate, in theory, given constant conditions, Florida would exceed Texas in about 543 years. However, since no state has unlimited capacity for growth, it is unrealistic to predict over a long period of time.

64. $\frac{\log 10^2}{\log 10^1} \neq \log 10^{2-1}$

65. Answers may vary. Sample: $\log x = 1.6$

$10^{1.6} = x, x \approx 39.81$

66. Answers may vary. Sample: A possible model is $y = 1465(1.0838)^x$ where $x = 0$ represents 1991; the growth is probably exponential and $1465(1.0838)^{10} \approx 3276$; using this model, there will be 10,000 manatees in about 2015.

67. a. $x = \frac{\log b}{\log a}$

b. $x = \log_a b = \frac{\log b}{\log a}$

c. Substituting the result from part (a) into the results from part (b), or vice versa, yields $\log_a b = \frac{\log b}{\log a}$. This justifies the Change of Base Formula.

68. $\frac{\log 2}{\log 7}$ **69.** $\frac{\log 8}{\log 3}$ **70.** $\frac{\log 140}{\log 5}$

71. $\frac{\log 3.3}{\log 9}$ **72.** $\frac{\log 3x}{\log 4}$ **73.** $\frac{\log (1 - x)}{\log 6}$

74. $\frac{\log 5}{\log x}$ **75.** $\frac{\log (x + 1)}{\log x}$

76. a. 10^0 (or 1) W/m^2, 10^4 W/m^2

b. 10,000 times more intense

77. a. top up: 10^{-5} W/m^2, top down: $10^{-2.5}$ W/m^2

b. 99.68%

78. a. 10^{-3} W/m^2, 106 W/m^2

b. 10^9 times more intense

79. 2.9315 **80.** 0.2098 **81.** 0.6225 **82.** 625

83. 2.3094 **84.** 10 **85.** 0.8505 **86.** 1.5

87. 7.4168 **88.** 200.8 **89.** 2.9615 **90.** 2.7944

91. 1 **92.** 500 **93.** 1.0451 **94.** $114.\overline{3}$

95. 1.3063 **96.** 3.0417

97. a. bassoon, guitar, harp, violin, viola, cello

b. bassoon, guitar, harp, cello, bass

c. harp, violin

d. harp, violin

98. 478,630 times

99. no; solving $0.65 = \frac{x}{(0.5)^{5430}}$ for x, the age in years of the sample, yields an age of about 3561 yrs.

100. 5 **101.** $-4, 2$ **102.** $-9, 9$ **103.** 1

104. 20,031 m above sea level

105. **a.** 91 hours or 4 days

b. 0.928 mg or 1.061 mg

c. Estimate in hours is more accurate; the days have a larger rounding error.

Answers for Lesson 8-6 Exercises

1. $\ln 125$ **2.** $\ln 18$ **3.** $\ln 4$

4. $\ln 40{,}960$ **5.** $\ln \frac{1}{81}$ **6.** $\ln 1$

7. $\ln \frac{m^5}{n^3}$ **8.** $\ln \frac{\sqrt[3]{xy}}{z^4}$ **9.** $\ln \frac{a\sqrt{c}}{b^2}$

10. 20.92 **11.** 24.13 **12.** 7.79 km/s; yes

13. 25 s **14.** 134.476 **15.** 0.135

16. 1.078×101^5 **17.** 1488.979 **18.** 5.482, -3.482

19. ± 11.588 **20.** 110.196 **21.** ± 2.241

22. ± 0.908 **23.** 2.890 **24.** 1.151

25. 2.401 **26.** 5.493 **27.** 1.242

28. 23.752 **29.** 6 years **30.** 78%

31. 1 **32.** 2 **33.** 10

34. 10 **35.** 0 **36.** $\frac{1}{4}$

37. 1 **38.** 83 **39.** 301

40. 10.8 **41.** sometimes

42. never **43.** always

44. about 5.8% per hour **45.** 19.8 h

46. about 40,000 bacteria **47.** 3.6

48. 6.7 **49.** 9.4 **50.** 11.8

51. 13.9 **52.** 15.8 **53.** 17.5

54. 19.1 **55.** 542.31 **56.** 1

57. 0.0794 **58.** 81.286 **59.** 1.2639

60. no solution **61.** 27,347.9 **62.** 78.342

63. No; using the Change of Base Formula would result in one of the log expressions being written as a quotient of logs, which couldn't then be combined with the other expression to form a single logarithm.

64. **a.** $y = 300e^{0.241t}$

b. 2002

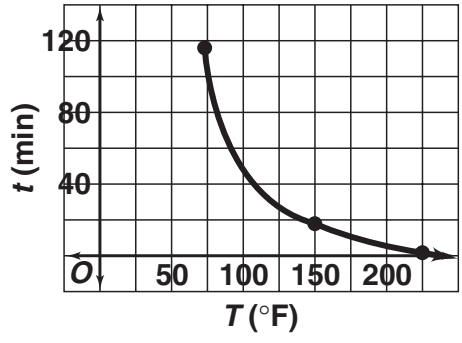
c. 2006

d. $t = \frac{\ln\left(\frac{y}{300}\right)}{0.241}$, where y is the number of Internet users in millions and t is time in years.

e. Substitute the number of users found in (b) and (c) into the equation in (d). Determine whether your answers in years are the same as t for each.

65. **a.** about 43 min

b. $t = \frac{-1}{0.041} \ln\left(\frac{T - 72}{164}\right)$



1.7, 6.0, 11.3, 18.1, 27.6, 43.1, 97.6

66. Check students' work.