

WARM UP WITH CALCULATOR

Solve for x:

1) $12 = 10^{x+5} - 7$

$$19 = 10^{x+5}$$

$$\log 19 = \log 10^{x+5}$$

$$\frac{\log 19}{\log 10} = \frac{(x+5) \log 10}{\log 10}$$

$$1.28 = x + 5$$

$$x = -3.72$$

2) $\log_3 (2x - 6) = 3$

$$3^3 = 2x - 6$$

$$27 = 2x - 6$$

$$33 = 2x$$

$$x = \frac{33}{2}$$

3) $9^{2x+1} = \left(\frac{1}{3}\right)^{3x}$

$$3^{2(2x+1)} = 3^{-1(3x)}$$

$$4x + 2 = -3x$$

$$2 = -7x$$

$$x = -\frac{2}{7}$$

4) You purchased a new car for \$24,900. It is depreciating at a rate of 10.3% per year. What will be the estimated value of the car in 4 year?

$$y = ab^x \quad r = .103$$

$$a = 24,900$$

$$b = 1 \pm r$$

$$24,900 (1 - .103)^4 = 16,120.15 \text{ dollars}$$

5) Condense: $5 \log_4 x + 3 \log_4 y - \log_4 z$

$$\log_4 x^5 + \log_4 y^3 - \log_4 z$$

$$\log_4 x^5 y^3 - \log_4 z$$

$$\log_4 \frac{x^5 y^3}{z}$$

HW 8.5 p. 465 Part II - ANSWERS

53. -1

54. 3

55. $\frac{1}{2}$

56. 3

57. $\frac{1}{3}$

58. -2

59. 3

60. $-\frac{1}{2}$

83. 2.3094

84. 10

85. 0.8505

86. 1.5

87. 7.4168

88. 200.8

89. 2.9615

90. 2.7944

91. 1

92. 500

93. 1.0451

94. $114.\bar{3}$

95. 1.3063

p. 468

96. 3.0417

1. 2. Graphs
3. $3 \log s - 5 \log r$
4. $2 \log_6 3 + 2 \log_6 x + 2 \log_6 y$
5. $\log_6 4 + \frac{1}{2} \log_6 x$

8.6 Natural Logarithms

For the equation $\log 100 = x$, what is the implied base?

If the base of a log is e , we use the \ln button on the calculator so,

\log_e is written \ln

$$\log_e = \ln$$

\ln is said to be "the natural log"

All properties of logs apply to the natural log.

Write as a single ln.

$$2 \ln 12 - \ln 9 \quad \text{Pull} \rightarrow$$

$$\ln 12^2 - \ln 9$$

$$\ln 144 - \ln 9 = \ln \frac{144}{9} = \ln 16$$

$$\left(\frac{1}{4}\right) \ln 3 + \left(\frac{1}{2}\right) \ln x - 5 \ln x \quad \text{Pull} \rightarrow$$

$$\ln 3^{1/4} + \ln x^{1/2}$$

$$\ln \sqrt[4]{3} + \ln \sqrt{x} - \ln x^5$$

$$\ln \sqrt[4]{3} \sqrt{x} - \ln x^5$$

$$\ln \frac{\sqrt[4]{3} \sqrt{x}}{x^5}$$

Solve for x.

$$\ln(3x - 9) = 5 \quad \text{Pull} \rightarrow$$

$$\log_e(3x - 9) = 5$$

$$e^5 = 3x - 9$$

$$\begin{array}{r} +9 \qquad +9 \\ \hline \end{array}$$

$$\frac{e^5 + 9}{3} = \frac{3x}{3}$$

$$x = \frac{e^5 + 9}{3} = 52.47$$



$$\ln \frac{x+2}{3} = 12 \quad \text{Pull} \rightarrow$$

$$\log_e \frac{x+2}{3} = 12$$

$$\downarrow$$

$$3 \cdot e^{12} = \frac{x+2}{3} \cdot 3$$

$$3e^{12} = x + 2$$

$$3e^{12} - 2 = x$$

$$x = 488,262.37$$

Solve for x.**Get e^x alone, then take \ln of each side.**

$$e^{2x} - 8 = 7 \quad \text{Pull} \rightarrow$$
$$\begin{array}{r} +8 \quad +8 \\ \hline \end{array}$$

$$e^{2x} = 15$$

$$\cancel{\ln e^{2x}} = \ln 15$$

$$\frac{2x}{2} = \frac{\ln 15}{2}$$

$$x = \frac{\ln 15}{2} = 1.35$$

$$4e^{3x} + 1.2 = 14 \quad \text{Pull} \rightarrow$$
$$\begin{array}{r} -1.2 \quad -1.2 \\ \hline \end{array}$$

$$\frac{4e^{3x}}{4} = \frac{12.8}{4}$$

$$e^{3x} = 3.2$$

$$\cancel{\ln e^{3x}} = \ln 3.2$$

$$\frac{3x}{3} = \frac{\ln 3.2}{3}$$

$$\boxed{x = .39}$$

HOMEWORK 8.6 - Part I

p. 472 #1-9 odd, 15-37 odd,
omit #29

Warm Up - Solve for x in each equation (#1-3).

1) $\ln (2x - 3) = 9$

2) $2 + \ln (x - 1)^2 = 7$

3) $e^{2x} - 8 = 7$

4) Evaluate:

$$\log_9 \frac{1}{3} + 3 \log_9 3 - \log_9 9$$

CHECK HW 8.6 Part 1 p. 472

1. $\ln 125$

2. $\ln 18$

3. $\ln 4$

4. $\ln 40,960$

5. $\ln \frac{1}{81}$

6. $\ln 1$

7. $\ln \frac{m^5}{n^3}$

8. $\ln \frac{\sqrt[3]{xy}}{z^4}$

9. $\ln \frac{a\sqrt{c}}{b^2}$

21. ± 2.241

31. 1

32. 2

33. 10

34. 10

35. 0

36. $\frac{1}{4}$

55. $x = 542.31$

56. $x = 1$

58. $x = 81.29$


**Word Problems
involving the
Natural Log**

What equation about growth do you know
which involves e ?

Solve the following problem.
Remember to use $A = Pe^{rt}$

An investment of \$100 is now valued at \$149.18. The interest rate is 8% compounded continuously. About how long has the money been invested?

Use $A = Pe^{rt}$ 

What variable in the equation are you trying to find? 

$$A = Pe^{rt}$$

 Pull

$$\frac{149.18}{100} = \frac{100e^{.08t}}{100}$$

$$1.4918 = e^{.08t}$$

$$\ln(1.4918) = \ln e^{.08t}$$

$$\frac{\ln(1.4918)}{.08} = \frac{.08t}{.08}$$

$$t = 5.0 \text{ years}$$

An initial investment of \$200 is worth \$315.24 after 7 years of continuous compounding. Find the interest rate.



$$A = Pe^{rt}$$

$$\frac{315.24}{200} = \frac{200e^{r(7)}}{200}$$



$$1.58 = e^{7r}$$

$$\ln(1.58) = \ln(e^{7r})$$

$$\ln(1.58) = 7r$$
$$.0645$$

$$\boxed{6.5\% = r}$$

The population of Boringville is 800 dull folks. The number of residents is decreasing at a rate of 32% per year. How many years will it take before the number of inhabitants is 250 people?

HOMEWORK 8.6 - Part2

P. 472 #2-8 even, 14-38 even, 29,
47-53 odd, 56-59 all

$$1. ab^x$$

$$b = 1 + r$$

$$1 - .023$$

$$3500(1 - .023)^7$$

$$1.05 \quad 5\%$$

$$.65 \quad .65 = 1 + r$$

$$- .35 = r$$

$$\log_{10} 10 = x$$

$$10^x = 10$$

$$(12) \log x^3 \sqrt{y} = \log x^3 + \log y^{1/2}$$

$$3 \log x + \frac{1}{2} \log y$$

$$(15) 4 \log x^3 + \log x^5$$

$$\log x^{12} + \log x^5$$

$$\log x^{12} x^5 = \log x^{17}$$

EXTRA PROBLEMS

1) You have inherited land that was purchased for \$20,000 in 1970. The value of the land increased by approximately 8% per year. What was the value of the land last year?

2) You purchased a new car for \$24,900. It is depreciating at a rate of 10.3% per year. What will be the estimated value of the car in 4 year?

3) Solve for x:

4) Expand: $\log \frac{x^5 y^{-2}}{2z}$

5) Condense: $5 \log_4 x + 3 \log_4 y - \log_4 z$

6) Evaluate: $\log_9 \frac{1}{3} + 3 \log_9 3 - \log_9 9$

Solve each equation. If necessary, round to the nearest ten-thousandth.

79. $8^x = 444$ **2.9315**

80. $14^{9x} = 146$ **0.2098**

81. $3^{7x} = 120$ **0.6225**

82. $\frac{1}{2} \log x + \log 4 = 2$ **625**

83. $4 \log_3 2 - 2 \log_3 x = 1$ **2.3094**

84. $\log x^2 = 2$ **10**

85. $9^{2x} = 42$ **0.8505**

86. $\log_8 (2x - 1) = \frac{1}{3}$ **1.5**

87. $1.3^x = 7$ **7.4168**

88. $\log (5x - 4) = 3$ **200.8**

89. $2.1^x = 9$ **2.9615**

90. $12^4 - x = 20$ **2.7944**

91. $5^{3x} = 125$ **1**

92. $\log 4 + 2 \log x = 6$ **500**

93. $4^{3x} = 77.2$ **1.0451**

94. $\log_7 3x = 3$ **114.3**

95. $3^x + 0.7 = 4.9$ **1.3063**

96. $7^x - 1 = 371$ **3.0417**

tial and Logarithmic Functions

Attachments

jnvu6kq1.bmp