WARM UP

1) Given, $f(x) = 4x$ and $g(x) = \frac{1}{2}x + 7$ find the following:

$(f \circ g)(x)$

$f(g(x)) = f(\bigstar) = 4(\bigstar)$

$= 4\left(\frac{1}{2}x + 7\right) = 2x + 28$

$(g \circ f)(x)$

$\frac{1}{2}(4x) + 7$

$2x + 7$

$f(g(-12))$

$g(-12) = \frac{1}{2}(-12) + 7 = 1$

$f(1) = 4(1) = 4$

2) Find the Domain:

$$f(x) = \frac{3x^2 + 4}{2x^2 + x - 3} = 0$$

-6

$3 \quad -2$

1

$(x + \frac{3}{2})(x - \frac{2}{2})$

$(2x + 3)(x - 1) = 0$

$x \neq -\frac{3}{2} \quad x \neq 1$

$\left(-\infty, -\frac{3}{2}\right)\left(-\frac{3}{2}, 1\right)(1, \infty)$

3) Find the Domain:

$$g(x) = -5\sqrt{\boxed{2x - 5}} + 1$$

$2x - 5 \geq 0$

$2x \geq 5$

$x \geq \frac{5}{2}$

$\cancel{\left(-\infty, \frac{5}{2}\right)}$ $\cancel{\left(\frac{5}{2}, \infty\right)}$

-3

$$\boxed{\left[\frac{5}{2}, \infty\right)}$$

7.7 Inverse Relations and Functions

If a relation pairs element x of its domain to element y of its range, the **inverse relation** pairs y with x.

inverse relation

(x, y) → (y, x)

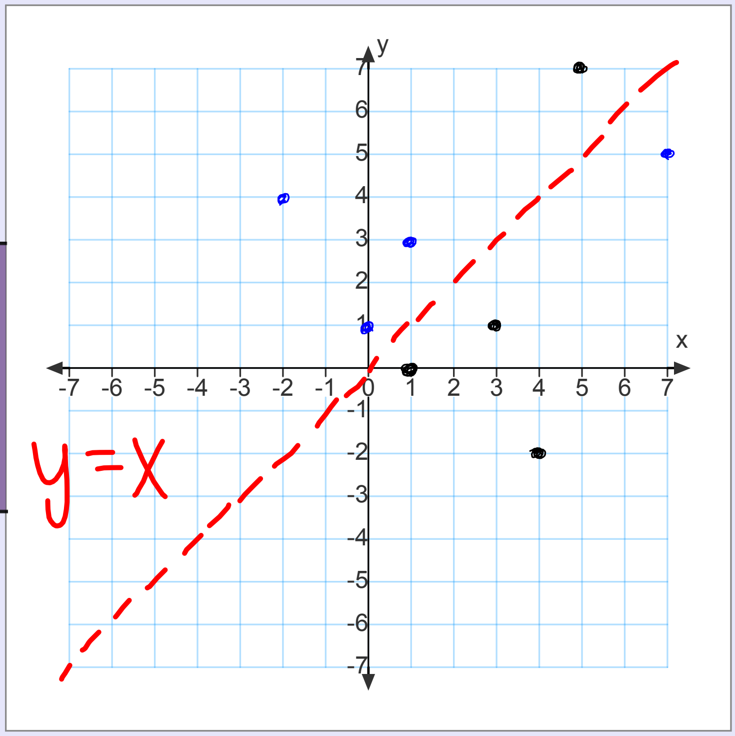
For example:

X	Y
1	0
3	1
4	-2
5	7

→

X	Y
0	1
1	3
-2	4
7	5

Pull



Get It?

The graph of the inverse of a relation is the **reflection over the line y = x** of the graph of the relation. If a relation or function is described by an equation in x and y, you can **interchange x and y to get the inverse**.

Ex. 1 Find the inverse of $y = x^2 + 4$.

Switch x and y.

Pull

$x = y^2 + 4$

P

$-4 \quad -4$

Solve for y.

Pull

E

$\sqrt{x-4} = \sqrt{y^2}$

MD

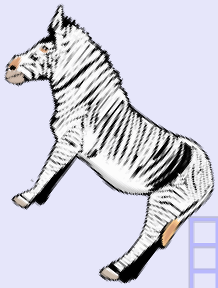
AS

$\boxed{\pm\sqrt{x-4} = y}$

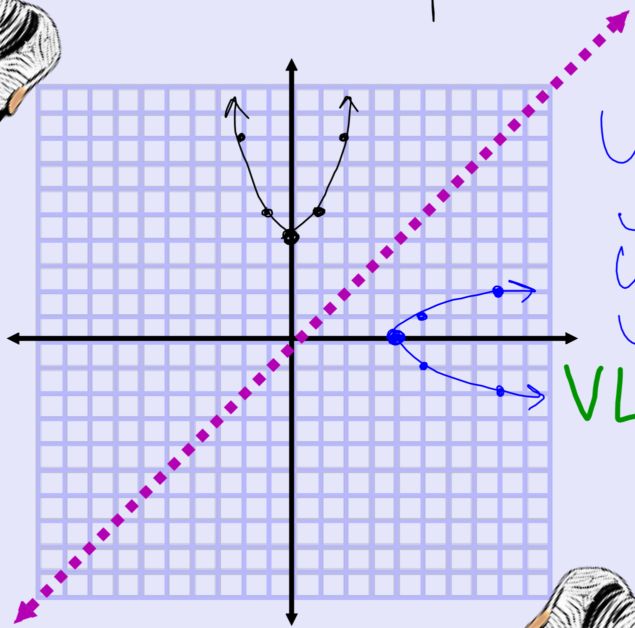
From the last example

a. Is $y = x^2 + 4$ a function?

b. Is the inverse a function? Explain.



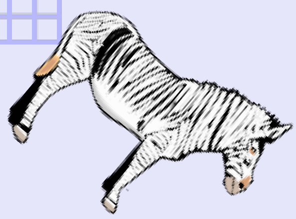
Graph $\boxed{y = x^2 + 4}$ and its inverse, $y = \boxed{\pm\sqrt{x-4}}$



$y = \sqrt{x-4}$

$y = -\sqrt{x-4}$

VLT



Example #2

a. Find the inverse of $\boxed{y = 2x - 10.}$

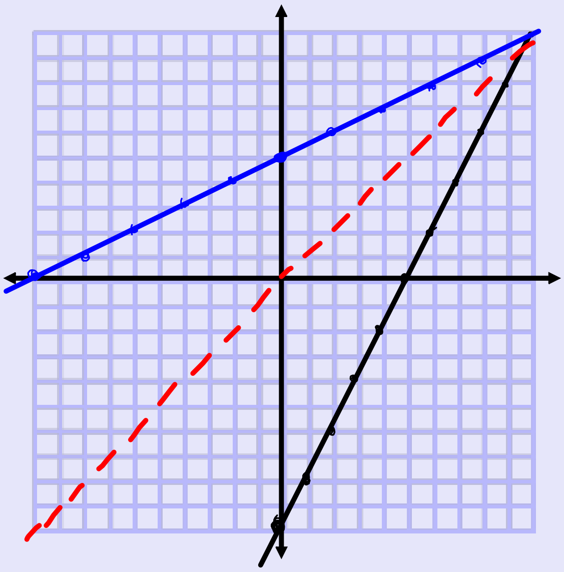
① $x = 2y - 10$ $\frac{x+10}{2} = \frac{2y}{2}$

$+10 \quad +10$

b. Is the inverse a function? Explain.

$\frac{x}{2} + 5 = y$

$\boxed{\frac{1}{2}x + 5 = y}$



Example #3:

a. Find the inverse of $y = (x - 1)^2 - 5$

b. Is the inverse a function? Explain.

The inverse of function f is denoted by f^{-1}.

We read f^{-1} as "the inverse of f" or as "f inverse."

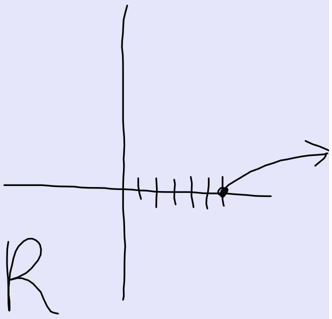
$f(x)$ is a function

$f^{-1}(x)$ may not be a function.

Example 5: **Let $f(x) = \sqrt{x-6}$**

y

a) Find the domain and range of $f(x)$



$x - 6 \geq 0$ $[6, \infty) = D$ $[0, \infty) = R$

$x^2 = 3$ $x \geq 6$

b) Find f^{-1}

$(x)^2 = (\sqrt{y-6})^2$; $x^2 = y - 6$; $x^2 + 6 = y^{-1}$

c) Find the domain and range of f^{-1}

D: $[0, \infty)$ R: $[6, \infty)$

d) Find $f^{-1}(f(15))$

$f(15) = \sqrt{15-6} = \sqrt{9} = \pm 3$

$f^{-1}(\pm 3) = x^2 + 6$

$3^2 + 6 = 9 + 6 = 15$

$15 \to \pm 3 \to 15$

If f and f^{-1} are both functions,
they are called
inverse functions,
then

$$\boxed{f^{-1}(f(x)) = x \text{ and } f(f^{-1}(x)) = x}$$

which can be written

$$\boxed{\left(f^{-1} \circ f\right)(x) = x \quad \text{and} \quad \left(f \circ f^{-1}\right)(x) = x}$$

Example #4:

For the function $f(x) = 2x + 10$ find:

y

$f^{-1}(x) =$

$x = 2y + 10$

$-10 \qquad -10$

$\frac{x}{2} - \frac{10}{2} = \frac{2y}{2}$

$\frac{1}{2}x - 5$

$(f^{-1} \circ f)(-4)$

$f^{-1}(f(-4))$

$2(-4) + 10$

$-8 + 10 = 2$

$f^{-1}(2) = \frac{1}{2}(2) - 5 = -4$

$(f \circ f^{-1})(-4)$

Homework

p. 410
#1-19 odd
23-28 all
31-34 all

Homework

p. 410
#1-19 odd
23-28 all
31-34 all