Chapter 5 Study Guide

Find the exact value of each expression.

1. $\cos\frac{7\pi}{12}$
2. $\sin\frac{17\pi}{12}$
3. $\cos -15°$
4. $\sin 10°\cos 80° + \cos 10°\sin 80°$
5. $\cos\frac{2\pi}{9}\cos\frac{\pi}{9} - \sin\frac{2\pi}{9}\sin\frac{\pi}{9}$
6. $\sin 60°\cos 15° + \cos 60°\sin 15°$

Find the exact value of $\sin 2x$ *and* $\cos 2x$ for each of the following.

7. $\sin x = \frac{1}{4}$ *x is in quadrant* I
8. $\cos x = -\frac{3}{5}$ *x ix in quadrant III*
9. $\tan x = -\frac{4}{5}$ *x is in quadrant II*

Find the exact value of the expression given that $\sin x = -\frac{2}{3}$ and $\cos y = \frac{1}{4}$ both in quadrant IV.

10. $\sin(x - y)$
11. $\cos(y - x)$
12. $\tan(x + y)$

Verify that each is an identity.

13. $\frac{\tan\theta\csc\theta}{\sec\theta} = 1$
14. $\frac{cot^2\theta - 1}{1 + cot^2\theta} = 1 - 2sin^2\theta$
15. $\frac{sin^2\theta - \cot\theta\tan\theta}{\cot\theta\sin\theta} = -\cos\theta$
16. $\frac{\cos\theta}{\sec\theta - \tan\theta} = 1 + \sin\theta$
17. $\tan\beta(\cot\beta + \tan\beta) = sec^2\beta$
18. $tan^2\alpha\, cos^2\alpha = 1 - cos^2\alpha$
19. $\csc x \sec x = \cot x + \tan x$
20. $cos^2 x + tan^2 x\, cos^2 x = 1$
21. $\tan\theta\sin\theta = \frac{1 - cos^2\theta}{\cos\theta}$
22. $\sin x(\csc x - \sin x) = cos^2 x$

Chapter 5 Study Guide

1) $\cos\frac{7\pi}{12} = \cos\left(\frac{\pi}{4}+\frac{\pi}{3}\right)$

$= \cos\frac{\pi}{4}\cos\frac{\pi}{3} - \sin\frac{\pi}{4}\sin\frac{\pi}{3}$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{1-\sqrt{3}}{2\sqrt{2}}$

2) $\sin\frac{17\pi}{12} = \sin\left(\frac{7\pi}{6}+\frac{\pi}{4}\right)$

$= \sin\frac{7\pi}{6}\cos\frac{\pi}{4} + \cos\frac{7\pi}{6}\sin\frac{\pi}{4}$

$= \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{-1-\sqrt{3}}{2\sqrt{2}}$

3) $\cos(-15)^\circ = \cos(45-60)$

$= \cos 45\cos 60 + \sin 45\sin 60$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{1+\sqrt{3}}{2\sqrt{2}}$

4) $\sin 10^\circ\cos 80^\circ + \cos 10^\circ\sin 80^\circ = \sin(10+80)$

$= \sin 90$

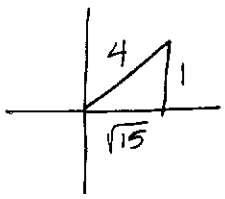
$= 1$

5) $\cos\frac{2\pi}{9}\cos\frac{\pi}{9} - \sin\frac{2\pi}{9}\sin\frac{\pi}{9} = \cos\left(\frac{2\pi}{9}+\frac{\pi}{9}\right)$

$= \cos\frac{\pi}{3}$

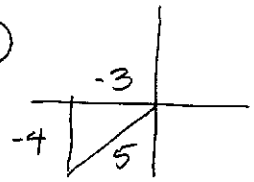
$= \frac{1}{2}$

6) $\sin 60\cos 15 + \cos 60\sin 15 = \sin 45 = \frac{1}{\sqrt{2}}$

7) 

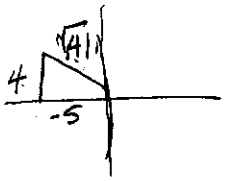
$\sin 2x = 2\sin x \cos x$
$= 2\left(\frac{1}{4}\right)\left(\frac{\sqrt{15}}{4}\right)$
$= \frac{2\sqrt{15}}{16}$
$= \frac{\sqrt{15}}{8}$

$\cos 2x = \cos^2 x - \sin^2 x$
$= \left(\frac{\sqrt{15}}{4}\right)^2 - \left(\frac{1}{4}\right)^2$
$= \frac{15}{16} - \frac{1}{16}$
$= \frac{14}{16} = 7/8$

8) 

$\sin 2x = 2\sin x \cos x$
$= 2(-4/5)(-3/5)$
$= \frac{24}{25}$

$\cos 2x = \cos^2 x - \sin^2 x$
$= \left(\frac{-3}{25}\right)^2 - \left(\frac{-4}{5}\right)^2$
$= \frac{9}{25} - \frac{16}{25}$
$= -7/25$

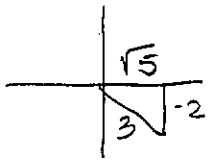
9) 

$\sin 2x = 2\sin x \cos y$
$= 2\left(\frac{4}{\sqrt{41}}\right)\left(\frac{-5}{\sqrt{41}}\right)$
$= \frac{-40}{41}$

$\cos 2x = \cos^2 x - \sin^2 x$
$= \left(\frac{-5}{\sqrt{41}}\right)^2 - \left(\frac{4}{\sqrt{41}}\right)^2$
$= \frac{25}{41} - \frac{16}{41}$
$= 9/41$

$\tan 2x = \frac{\frac{-40}{41}}{9/41} = -40/9$

$\sin x = -2/3$



x

$\cos y = 1/4$

y

10) $\sin(x-y) = \sin x \cos y - \cos x \sin y$
$(-2/3)(1/4) - (\sqrt{5}/3)\left(\frac{-\sqrt{15}}{4}\right)$
$\frac{-2}{12} + \frac{\sqrt{75}}{12}$
$\frac{-2+5\sqrt{3}}{12}$

11) $\cos(y-x) = \cos y \cos x + \sin y \sin x$

$= \left(\frac{1}{4}\right)\left(\frac{\sqrt{5}}{3}\right) + \left(\frac{-\sqrt{15}}{4}\right)\left(-\frac{2}{3}\right)$

$= \frac{\sqrt{5}}{12} + \frac{2\sqrt{15}}{12}$

$= \frac{\sqrt{5} + 2\sqrt{5}}{12}$

12) $\tan(x+y) = \frac{\sin(x+y)}{\cos(x+y)} = \frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y - \sin x \sin y}$

$= \frac{\left(-\frac{2}{3}\right)\left(\frac{1}{4}\right) + \left(\frac{\sqrt{5}}{3}\right)\left(\frac{-\sqrt{15}}{4}\right)}{\left(\frac{\sqrt{5}}{3}\right)\left(\frac{1}{4}\right) - \left(-\frac{2}{3}\right)\left(\frac{\sqrt{15}}{4}\right)}$

$= \frac{\frac{-2-\sqrt{75}}{12}}{\frac{\sqrt{5}-2\sqrt{15}}{12}} = \frac{-2-5\sqrt{3}}{\sqrt{5}-2\sqrt{15}}$

13) $\frac{\tan\theta \csc\theta}{\sec\theta} = 1$

$\frac{\frac{\sin\theta}{\cos\theta}\,\frac{1}{\sin\theta}}{\frac{1}{\cos\theta}}$

1

14) $\frac{\cot^2\theta - 1}{1+\cot^2\theta} = 1 - 2\sin^2\theta$

$\frac{\cot^2\theta - 1}{\csc^2\theta}$

$\frac{\cot^2\theta}{\csc^2\theta} - \frac{1}{\csc^2\theta}$

$\frac{\cos^2\theta}{\sin^2\theta} \cdot \frac{\sin^2\theta}{1} - \frac{1}{\csc^2\theta}$

$\cos^2\theta - \sin^2\theta$

$1 - \sin^2\theta - \sin^2\theta$

15) $\frac{\sin^2\theta - \cot\theta\tan\theta}{\cot\theta\sin\theta} = -\cos\theta$

$\frac{\sin^2\theta - 1}{\cot\theta\sin\theta}$

$-\frac{\cos^2\theta}{\cot\theta\sin\theta}$

$-\frac{\cos^2\theta}{\frac{\cos\theta}{\sin\theta}\sin\theta}$

$-\cos\theta$

16) $\frac{\cos\theta}{\sec\theta - \tan\theta} = 1 + \sin\theta$

$\frac{\cos\theta}{\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}}$

$\cos\theta \cdot \frac{\cos\theta}{1-\sin\theta}$

$\frac{\cos^2\theta}{1-\sin\theta} = \frac{1-\sin^2\theta}{1-\sin\theta} = \frac{(1-\sin\theta)(1+\sin\theta)}{1-\sin\theta}$

17) $\tan\beta(\cot\beta + \tan\beta) = \sec^2\beta$

$1 + \tan^2\beta$

$\sec^2\beta$

18) $\tan^2\alpha\cos^2\alpha = 1 - \cos^2\alpha$

$\frac{\sin^2\alpha}{\cos^2\alpha} \cdot \frac{\cos^2\alpha}{1}$

$\sin^2\alpha$

$1 - \cos^2\alpha$

19) $\csc x \sec x = \cot x + \tan x$

$\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}$

$\frac{\cos^2 x + \sin^2 x}{\sin x \cos x}$

$\frac{1}{\sin x \cos x}$

$\sec x \csc x$

20) $\cos^2 x + \tan^2 x \cos^2 x = 1$

$\cos^2 x (1 + \tan^2 x)$

$\cos^2 x (\sec^2 x)$

1

21) $\tan\theta\sin\theta = \frac{1 - \cos^2\theta}{\cos\theta}$

$= \frac{\sin^2\theta}{\cos\theta}$

$= \frac{\sin\theta}{\cos\theta} \cdot \sin\theta$

$= \tan\theta\sin\theta$

22) $\sin x(\csc x - \sin x) = \cos^2 x$

$1 - \sin^2 x$

$\cos^2 x$

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